Edm. John Senkler
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CAMBRIDGE PROBLEMS

PROPOSED BY THE MODERATORS

TO THE

CANDIDATES FOR MATHEMATICAL HONORS

AT THE

GENERAL EXAMINATIONS

FROM 1821 TO 1830 INCLUSIVE.

WITH AN INDEX OF THE SUBJECTS.

CAMBRIDGE:

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CAMBRIDGE PROBLEMS.

JANUARY, 1821.

MONDAY MORNING.—MR. CHEVALLIER.

FIRST AND SECOND CLASSES.

1. The number 803 expressed in a different scale of notation becomes 30203; required the radix of the scale.

2. Of all sections made by planes passing through both sides of an oblique cone, two are circles, and all the rest ellipses.

3. Why does the apparent distance of two fixed stars increase as they approach the horizon?

4. A clepsydra is constructed to mark equal portions of time, in the form of a paraboloid having its vertex downwards, the equation to the generating curve being $y^4 = ax$. How must the scale on the axis be graduated?

5. a and b are the arms of a straight lever which turns on an axis, the radius of which is r. P would maintain the equilibrium acting prependicularly at the distance (a), if there were no friction: but a weight p must be added to it in order to overcome the friction. Find the proportion of the friction to the pressure.

6. S being the focus of an hyperbola, and (pm) the perpendicular upon its directrix from a point (p) in the *opposite hyperbola*, $Sp : pm :: SC : AC$, a given ratio.

7. Find the law of force by which a body may describe a rectangular hyperbola, the force acting in parallel lines perpendicular to one of its asymptotes.

8. Two pendulums, the lengths of which are L and l, begin to oscillate together, and are again coincident after n oscillations of the first pendulum. Given L to find l.

9. The Moon revolves in a circle about the Earth, and the quantity of matter in the Earth is suddenly doubled. Compare the eccentricity of the orbit now described with its axis-major, and with the original radius of the Moon's orbit.

10. Shew that the hour of Sun-rise and Sun-set together = 12 hours nearly: and find the correction necessary if the Sun's declination should have changed by a given quantity.

11. Explain the construction of the steam engine; and having given the weight upon the piston, the quantity of steam admitted, and the content of the cylinder, find the velocity of the piston at any point, and the time of describing the cylinder.

12. Prove that

$$n^n - n.(n-1)^n + n\,\frac{n-1}{2}.(n-2)^n - \&c. = 1.2.3\ldots.n.$$

13. Find the force of the Sun to disturb the motions of the Moon.

MONDAY AFTERNOON.—Mr. Chevallier.

Fifth and Sixth Classes.

1. Find x from the equation

$$\sqrt{x} + \sqrt{a+x} = \frac{2a}{\sqrt{a+x}}.$$

2. The common difference of 4 numbers in arithmetical progression is 1, and their product 120; find the numbers.

3. A pendulum vibrating seconds at the earth's surface is

carried to a distance from the centre of the earth equal to that of the moon. What is the time of its vibration?

4. Shew that the image of a straight line placed between the center and principal focus of a concave mirror is an hyperbola.

5. In latitude 45°, find the time of sun-rise on the longest day.

6. Insert three harmonic means between a and b.

7. Compare the pressure on the surface of a sphere filled with water, with the weight of a sphere of mercury of the same magnitude.

8. A beam 30 feet long, balances itself upon a point at $\frac{1}{3}$rd of its length from the thicker end. But when a weight of 10lbs. is suspended at the other end, the prop must be moved two feet towards it to maintain the equilibrium. What is the weight of the beam?

9. Prove Demoivre's formula

$$(\cos A \pm \sqrt{-1}\,.\,\sin A)^m = \cos m\,A \pm \sqrt{-1}\,.\,\sin m\,A.$$

10. Given the three angles of a spherical triangle, to find its surface.

11. Sum the series

$$1 + 1 + \frac{3}{4} + \frac{1}{2} + \frac{5}{16} + \frac{3}{16} + \&\text{c. to } n \text{ terms.}$$

$$\frac{1}{1\,.\,3} + \frac{1}{3\,.\,5} + \frac{1}{5\,.\,7} + \&\text{c. to } n \text{ terms,}$$

by the method of increments.

12. Solve $x^3 - \frac{5}{2}x^2 + x - \frac{1}{6} = 0$, the roots being in harmonical progression.

13. Find the area of a curve in which the abscissa $= \theta$, and the ordinate $= \dfrac{\cos^2\theta}{r}$, between the values of $\theta = 0$, and $\theta = 90°$.

14. Find $d\left\{\frac{1}{\sqrt{-1}}\cdot\left(x\sqrt{-1}+\sqrt{1-x^2}\right)\right\}$.

$$\int\frac{dx\sqrt{2ax+x^2}}{x}.$$

$$\int\frac{dx}{(1+x^2)^3}.$$

15. Explain how wheels assist the motion of a carriage.

16. In what direction must a body be projected with a given velocity from a point in a given inclined plane, that the range may be the greatest possible?

17. Explain Archimedes' screw.

17. Find the time in which the moon would fall to the earth, if suddenly deprived of its angular motion.

MONDAY AFTERNOON.—Mr. Peacock.

Third and Fourth Classes.

1. In the solution of right-angled spherical triangles by Napier's rules, what cases are ambiguous?

2. The series

$$1-\frac{3}{x}+\frac{5}{x^2}-\frac{7}{x^3}+\frac{9}{x^4}-\text{\&c.}$$

is a recurring series: find its scale of relation and its sum *in infinitum*.

3. Investigate expressions for determining the position of the centre of gravity of a plane surface, bounded by a curve whose equation is given.

4. Two bodies are projected from two given points in given directions and with given velocities: find their distance at the end of t''.

5. Find the centre of pressure of a trapezoidal plane surface immersed vertically in a fluid, two of whose sides are parallel to each other, and to the surface of the fluid.

6. Shew how to find the focal length of a double concave lens by experiment.

7. Explain the method of determining accurately the obliquity of the ecliptic: to what inequalities is it subject? and from what causes do they arise?

8. Given the focal length and aperture of a Herschelian telescope: during what time will the image of a given star be visible in the tube?

9. Explain fully what is meant by the term *mean* in Astronomy.

10. In the 11th section of Newton, the mean motion of the nodes or apsides, varies as the periodic time of P directly, and as the square of the periodic time of T inversely.

11. A rod is placed in an inclined position, with one end upon a perfectly smooth horizontal plane: find the equation of the curve described by the other extremity whilst it falls.

12. Of all triangles upon equal bases and with equal vertical angles, the isosceles has the greatest perimeter.

13. Investigate a differential expression for the radius of curvature of a curve, referred to rectangular co-ordinates.

MONDAY EVENING.—Mr. Peacock.

1. Investigate the rule for transposing a number from one scale of notation to another.

2. The hypothenuse of a right-angled triangle, whether plane or spherical, being supposed invariable, to compute the corresponding variations of the two sides.

3. If, through a given point within a sphere, three planes pass, each of which is at right angles to the other two, the sum of the areas of the sections of the sphere, is a given quantity.

4. Explain what is meant by *accelerating force;* and when P draws up Q in a machine, find the force accelerating Q's ascent, both when P and Q move with the same and with different velocities, the inertia and friction of the parts of the machine not being considered.

5. If a body be balanced upon a horizontal plane, and a slight motion be given to it, its centre of gravity will move horizontally: prove this, and shew in what cases the equilibrium is stable.

6. If the air near the surface of the ground be less dense than at small altitudes above it, there will be observed inverted images of distant horizontal objects.

7. Explain what is meant by the Equation of Time, and from what causes it arises: at what times of the year is it nothing, and at what times is its negative and positive value respectively a *maximum*?

8. Find the equation of a straight line, which shall pass through two points whose co-ordinates are given.

9. Investigate the differential expression for the subtangent of a curve: and mention the analytical characters of a double, triple, and conjugate point.

10. Why is Cardan's formula for the solution of a cubic equation inapplicable when all the roots are possible? solve the equation in this case by trigonometrical formulæ, and reduce the results for logarithmic computation.

11. If we denote the sum, sum of the squares, cubes, .. n^{th} powers of the m quantities a, b, c, d, &c., by S_1, S_2, S_3, .. S_n, then the sum of the n^{th} powers of the differences of a, b, c, d, &c. or

$$(a-b)^n + (b-a)^n + (a-c)^n + \text{\&c.}$$

$$= m S_n - n S_1 S_{n-1} + \frac{n(n-1)}{1 \cdot 2} S_2 S_{n-2} - \text{\&c.}$$

12. If

$$\frac{P}{(x+a)^n Q} = \frac{A}{(x+a)^n} + \frac{A_1}{(x+a)^{n-1}} \dots + \frac{A_{n-1}}{(x+a)} + \frac{P'}{Q}$$

where P and Q are rational functions of x, to determine the values of A, A_1, $A_2 \dots A_{n-1}$ and P'.

13. If a be the arc of a circle whose radius is unity, a', a'' a''', &c. the arcs of its successive involutes, then

$$a + a' + a'' + a''' + \dots \textit{in infinitum} = e^a - 1.$$

14. Integrate the equations

$$(1)\quad \frac{dx}{1+x+x^2} + \frac{dy}{1+y+y^2} = 0.$$

$$(2)\quad y = \frac{x\,du^2}{dx^2} + \frac{dy}{dx}.$$

$$(3)\quad \frac{d^2 z}{dx^2} + \frac{3\,d^2 z}{dx\,dy} + \frac{2\,d^2 z}{dy^2} = x + y.$$

15. The curve which is expressed by the particular solution of a differential equation of the first order, is the locus of the intersections of the curves which arise from giving every possible value to the constant in the general solution.

16. A tetrahedron, whose base is an equilateral triangle, the side of which is equal to one half of each of the remaining edges, is thrown upon a horizontal table: what is the probability of its resting upon its base, excluding all consideration of the mechanical action, which arises from the rotation of the solid?

17. Supposing γ Draconis to be affected by a sensible annual parallax, in what manner will its apparent place be affected both by aberration and parallax on March 20, June 21, September 23, and December 23, its right ascension being 18 hours nearly?

18. The level of a transit instrument is suspended by hooks from its axis: in what manner are the errors which arise from the axis not being horizontal, or from the level not being

parallel to the axis, distinguished from each other, and by what adjustments are they corrected?

19. A ray of light, after being refracted through media possessing *equal dispersive* powers, will always appear coloured at its emergence, unless the incident and emergent rays are parallel.

20. Two equal bodies A and B are connected by a string of given length: A is placed in a horizontal groove, and B hangs freely down, the string passing through an aperture which is continued along the bottom of the groove: a given velocity is given to A; find the position of B at the end of t''.

21. A cylinder of given length, is pressed down in a vertical position into a fluid, so that its upper end is on a level with the surface, the specific gravity of the cylinder being one half that of the fluid: the pressure being removed, to find the greatest height to which the upper end of the cylinder will rise above the surface of the fluid.

22. Find an expression for that part of the force of Jupiter to disturb Saturn, which acts in the direction of a tangent to Saturn's orbit.

23. If the accelerating gravity of a satellite of Jupiter to the Sun was greater than that of Jupiter at the same distance, in the ratio of $e : 1$, where e differs little from unity, then the distance of the centres of the Sun and of the orbit of the satellite, would be greater than the distance of the centres of the Sun and Jupiter in the ratio of $\sqrt{e} : 1$.

24. If the Sun and Moon be both supposed to be in the equator: to compare the lengths of a lunar and a tide day, for a given position of the luminaries.

25. Shew that all the sections of an ellipsoid made by parallel planes are similar ellipses.

TUESDAY MORNING.—Mr. Peacock.

First and Second Classes.

1. If a and b be prime numbers, the number of numbers prime to $a\,b$ and less than $a\,b$, is equal to

$$(a-1)(b-1),$$

unity being considered as one of them.

2. In what cases are solid figures said to be similar and equal, according to Euclid? to what objections are these definitions liable?

3. Mention some of the experiments and observations, from which we may infer the truth of the second law of motion.

4. A given globe *rolls* down a given inclined plane in a medium resisting as the square of the velocity; to find the time of describing a given space.

5. Find the equation of the curve traced out by the extremities of the perpendiculars upon the tangents of a circle, drawn from a point in its circumference; and find its greatest ordinate. N.73.

6. An object appears brighter, *cæteris paribus*, when seen through a convex lens, than when seen through a concave lens.

7. Explain the method of determining accurately, when the first point of Aries is upon the meridian.

8. Explain the method of determining the sun's meridian altitude by means of a sextant: 1st. on the open sea: 2d. when the sun's altitude is not less than 60°, but a neighbouring coast is on the same side of the ship with the sun: 3dly. on land.

9. In elliptical orbits of small eccentricity, the diminution of angular velocity in moving from the lower apse to the higher, is nearly proportional to the increase of distance.

10. A degree of latitude in latitude 45°, is nearly an arithmetic mean between a degree at the equator and the pole.

11. Given the logarithm of n, to find the logarithm of $n+1$.

12. The integration of the partial differential equation
$$P p + Q q = R,$$
where $p = \frac{d z}{d x}$ and $q = \frac{d z}{d y}$, and P, Q and R are functions of the variables x, y and z, is reduced to the integration of equations of two variables, when any one of the equations
$$P\, d y - Q\, d x = 0,$$
$$P\, d z - R\, d x = 0,$$
$$Q\, d z - R\, d y = 0,$$
involves two variables only.

13. Two bodies A and B are placed upon a horizontal plane, and connected by a rigid rod without weight: a body C impinges upon a given point of the rod, in a given direction and with a given velocity: define the motions of A and B, the body C not being connected with the system after the impact.

14. The attraction of a spherical shell upon a particle placed without it, is the same as if the whole matter in the shell were placed in its centre.

TUESDAY AFTERNOON.—Mr. Peacock.

Fifth and Sixth Classes.

1. A person paid a tax of 10 per cent. upon his income: what must his income have been, when after he had paid the tax, he had £1250. remaining?

2. Prove the rule for extracting the square root in numbers.

3. The difference between any number and that number inverted, is divisible by 9.

4. If two straight lines be at right angles to the same plane, they are parallel to one another.

5. Given two sides and the included angle of a plane triangle: find the remaining parts, and reduce the results to logarithmic computation.

6. Every equation has at least as many changes of sign from + to – and from – to +, as it has positive and possible roots; and as many continuations of sign from + to +, and from – to –, as it has negative and possible roots.

7. Two tangents to a parabola drawn from the same point of the directrix, are at right angles to each other.

8. Prove that the sides of the polar or supplemental triangle are supplements of the angles of the given triangle.

9. When a body is uniformly accelerated from rest, to find the space described in a given time.

10. Shew, when P sustains W upon a screw, if a slight motion be given to the machine, that P's velocity : W's velocity :: W : P.

11. Distinguish between the centre of gravity and centre of pressure, and shew that the former is always nearer to the surface of the fluid than the latter.

12. Graduate a thermometer according to Fahrenheit's scale.

13. Prove that objects appear erect in Galileo's telescope.

14. By what experiments is it proved, that light consists of rays differing in colour and refrangibility?

15. What are the principal phænomena presented by the Sun and Earth in the course of a month, to a lunar observer?

16. Explain the mode of constructing a catalogue of the fixed stars.

17. When is the planet Venus stationary, and when retrograde?

18. A body describes an ellipse, the force being in the centre: given the force at a given distance, to find the actual periodic time.

19. A body is projected from a given point, in a given direction with a given velocity, when the force varies inversely as $(\text{dist.})^2$: find the latus-rectum of the orbit described.

20. Investigate the differential expression for the length of a curve.

21. Trace the curve whose equation is

$$x^4 - a^2 x^2 + a^3 y = 0,$$

and find its points of contrary flexure.

22. Find the integrals of

$$\frac{dx}{\sqrt{(a - b x^2)}}, \quad \frac{x\, dx}{(x + a)(x + b)},$$

$$d\theta (\sin\theta)^3 \qquad a^x x^3\, dx.$$

23. Prove that

$$\log(1 + u) = u - \frac{u^2}{2} + \frac{u^3}{3} - \frac{u^4}{4} + \&c.$$

24. Two equal weights are fixed, one at the middle point, and the other at the extremity of an inflexible and imponderable rod, which is suspended at the other extremity: if this compound pendulum be made to vibrate through small arcs, to find the time of its vibrations.

TUESDAY AFTERNOON.—Mr. Chevallier.

Third and Fourth Classes.

1. Shew how the logarithms of the natural numbers from 1 to 12 may be computed.

2. The sides of a plane triangle are 3, 5, 6: compare the radii of the inscribed and circumscribed circles.

3. If two straight lines intersect each other in a circle, the sum of the arcs cut off between their extremities is the same as that cut off by any two lines respectively parallel to them, and intersecting each other within the circle. Prove this property, and shew its use in correcting observations made with circular instruments inaccurately centered.

4. If a ladder slides down a perpendicular wall, shew that each stave describes a quadrant of an ellipse, except the middle one, which describes a quadrant of a circle.

5. Apply D'Alembert's principle to find the velocity and time when p draws q over a fixed pulley.

6. Explain the phases of the earth as they would be seen from the moon.

7. A small aperture (a) is made in the vertical side of a cylindrical vessel filled with a fluid; the area of its horizontal section being A. Compare the latus-rectum of the parabola first described by the spouting fluid with the length of a pendulum vibrating once while the surface of the fluid descends to the orifice.

8. The earth being a sphere, and its radius 4000 miles, what must be its diurnal rotation that a body at the equator may lose half its weight?

9. A sphere of given radius is suspended by a point at a distance from its centre equal to its diameter. Find the time of its oscillation, and the point within the sphere, at which it must be suspended so as to oscillate in the same time.

10. In a revolving spheroid of small eccentricity, if polar gravity : equatoreal sensible gravity : radius of equator : semi-axis, gravity is every where perpendicular to the spheroidal surface.

11. Apply the duodenary scale of notation to find the solidity of a cube the side of which is $13^{f}.\ 7^{in}.\ 7^{pts}$.

12. On board a ship in north latitude, Jupiter is observed on the meridian at $3^{h}.\ 4^{m}.\ 56^{s}$. and his corrected altitude is $29°.\ 6'.\ 42''$. One of his satellites is at the same instant eclipsed. His tabulated declination is $5°.\ 4'.\ 35''$. north, and the tabulated time of the eclipse $7^{h}.\ 0^{m}.\ 32^{s}$. Required the latitude and longitude of the ship.

TUESDAY EVENING.—Mr. Chevallier.

1. Prove the rule for the multiplication of duodecimals.

2. Represent $\sqrt{2n\sqrt{-1}}$ as a binomial surd.

3. Find two numbers such that their sum, product, and the difference of their squares may be all equal.

4. How many different ways may £100. be paid in crowns and guineas?

5. A body describes a logarithmic spiral, and approaches the centre by a space which is small compared with the whole distance. Compare the time of one revolution with the time to the center.

6. In the expansion of $(a + b + c + \&c.)^w$, where $w = p + q + r + \&c.$; find the co-efficient of the term involving $a^p . b^q . c^r . \&c.$

7. Shew that the image of an indefinite straight line perpendicular to the axis of a convex lens, and nearer to its center than the principal focus of parallel rays incident in the opposite direction, forms the arcs of two opposite hyperbolas; and find the semi-axes.

8. Explain *fully* the method of finding the longitude at sea by the observed distance between the Moon and a Star and their altitudes.

9. A vessel is kept filled with a fluid; and an aperture is made in its perpendicular side in the form of a parabola, the vertex of which coincides with the surface of the fluid. Find the depth of a horizontal section such that if the whole fluid issued with *its* velocity, the quantity discharged in a given time would be the same as when each horizontal section flows with its own velocity.

10. A recurring series may generally be resolved into two or more geometric series.

11. Find the solid content of a sphere by referring it to three rectangular co-ordinates.

12. Shew, by measuring the area ANB in the 12th section, that if a sphere be composed of particles the attraction of which $\propto \frac{1}{(\text{dist.})^2}$, the attraction of the whole sphere on an external particle varies in the same law.

13. At what distance from its center must the Earth, considered as a sphere, receive a single impulse, so as to produce its diurnal and annual rotation?

14. A body oscillates in a cycloid: compare the whole tension of the string at any point with the weight of the body.

15. Give Clairaut's approximation to the solution of a cubic equation in the irreducible case.

16. Trace the curve the equation to which is

$$xy + ay + bx = c.$$

17. Sum the series,

$$1^2 - 2^2 + 3^2 - 4^2 + \&c. \pm x^2.$$

18. A comet describes 90° from the perihelion in 100 days. Compare its perihelion distance with the radius of a planet's circular orbit which revolves about the Sun in 942 days.

19. Explain Borda's circle of repetition: and the method of finding the latitude by the zenith distances of stars near the meridian.

20. Find $\Delta^3 . u_x$, $\Sigma . x^3$, and $\Sigma . \sin x\theta$.

21. Integrate

$$\frac{dx}{\sqrt{1+x^2}} + a\,dx + 2by\,dy = 0. \quad \frac{d^4 y}{dx^4} = \frac{d^2 y}{dx^2}.$$

22. If a body is projected in any direction, and acted upon continually by two forces tending to fixed centers, not both in the same plane with the direction of projection, it will describe by lines drawn from the two fixed points equal solids in equal times about the line joining the two points.

23. Given, in the equation $\frac{d^2 u}{d v^2} + u = 0$, $u = a \sin v + b . \cos v$, to solve the equation $\frac{d^2 u}{d v^2} + u + \mathrm{H} = 0$, by the *Variation of the Parameters.*

24. A body oscillates in a cycloid, in a medium the resistance of which $\propto$ (vel.)^2; construct for the resistance at any point.

1822.

MONDAY MORNING.—Mr. Chevallier.

First and Second Classes.

1. Find a series of fractions converging to $\sqrt{17}$.

2. If $ABCD$ is a parallelogram and AC a diameter, and from B there be drawn a straight line cutting the diameter in E and the two sides, or the sides produced, in F, f respectively; shew that $EF \cdot Ef = BE^2$.

3. If $1, \rho, \rho^2, \ldots\ldots \rho^{n-1}$ are the roots of the equation $x^n - 1 = 0$; find the value of

$$1^m \cdot \rho^r + \rho^m \cdot 1^r + \&c. + \rho^m \cdot \rho^{2r} + \rho^{2m} \cdot \rho^r + \&c.$$

4. Integrate the quantities

$$\frac{dx}{x^n\sqrt{1+x^2}}, \; n \text{ being an even number}; \; \frac{dx}{x^2\sqrt{a+bx+cx^2}}.$$

5. What must be the solid of revolution, so that when suspended by its vertex the centre of oscillation may be in its base?

6. If a triangular prismatic beam is supported at both ends, shew that it is twice as strong when the edge is uppermost, as when the base is.

7. A person wishes to see distinctly when under water. What kind of glasses must he use, and of what focal length?

8. The autumnal equinox takes place at 6 in the evening, the Moon being full at the same instant, and in her ascending node. The next night the Moon rises at the *same hour*: required the north latitude of the place.

9. Given the base of a plane triangle and the difference of the angles at the base, to find the curve traced by the vertex.

10. Two equal heavy balls are suspended, by wires of the same given length, from the vertical axis of a machine and are just in contact. How far will they separate from one another when a given angular velocity is communicated to the system?

11. Shew that if M and S represent the height of the tide produced by the Moon and Sun respectively; retardation of tide at new and full Moon : retardation in quadratures :: $M - S : M + S$.

12. Find the mean horary motion of the Moon's nodes, when the line of the nodes is in octants.

MONDAY AFTERNOON.—Mr. Chevallier.

Fifth and Sixth Classes.

1. The first term of a geometric series continued *in infinitum* is 1, and any term is equal to the sum of all the succeeding terms. Required the series.

2. Transform $x^3 - 2x^2 + 2x - 4 = 0$ into an equation, the roots of which are the squares of the roots of the original equation.

3. Sum the series

$$\frac{1}{2.4.6} + \frac{1}{4.6.8} + \text{\&c. to } n \text{ terms and } in\ infinitum.$$

4. Trace the curve, the equation to which is $y^2 - ax - ab = 0$. Draw a tangent to it at any point, and determine the angle at which the curve cuts the axis.

5. In a spherical triangle, having given two angles and the included side, it is required to find the other angle.

6. Given

$$\left.\begin{array}{l}\log 6753 = 3.8294967 \\ \log 6754 = 3.8295611\end{array}\right\} \text{to find } \log .67532.$$

7. Shew how the true value of a fraction may be found, the numerator and denominator of which both vanish upon assigning a particular value to the variable quantity, and find the value of

$$\frac{\tan x + \sec x - 1}{1 + \tan x - \sec x} \text{ when } x = 0.$$

8. Find the radius of curvature at any point of the catenary.

9. A body falls down a given inclined plane, and, at the instant that it begins to fall, another is projected upwards from the bottom of the plane with a velocity equal to that acquired in falling through n times its length. Where will they meet?

10. Find the position of the centre of gravity of the quadrant of a circular area.

11. An isosceles triangle is immersed perpendicularly in a fluid with its vertex coincident with the surface and its base parallel to it. How must it be divided by a line parallel to the base, so that the pressure upon the upper and lower parts respectively may be in the ratio of 1 : 7?

12. When the same string passes over any number of pullies and the parts of the string supporting any pulley at the lower block are not parallel to one another, find the proportion between P and W in equilibrio.

13. A hollow cylinder is viewed by an eye placed in its axis produced. Compare its apparent capacity when empty and when filled with water.

14. By what methods may the variation of the compass be determined: and to what point does the true north correspond when the variation is 22° 30′ west?

15. Find the latitude of the place in which the longest day contains 16 hours.

16. A body is projected in a given direction with a given velocity from a given point, and is acted upon by a *repulsive*

force, which varies as the distance from another given point. Required the curve which it will describe.

17. How may the phenomena of the Trade Winds be explained?

18. A bucket descends into a well, unwinding a string from a cylinder of given weight and radius. What is the velocity acquired in falling through a given space, and the time of descent, the weight of the string being neglected?

19. Integrate the quantities

$$\frac{dx}{(1-x^2)^{\frac{3}{2}}}, \quad \frac{dx}{a-bx^2}, \quad \frac{dx}{x^3-3x^2+2x}.$$

20. A body describes a circle about a fixed point, the force varying inversely as the square of the distance: another body, the attractive force of which varies in the same law, is introduced into the system. How will this affect the velocity of the body, the form of the orbit and the periodic time?

MONDAY AFTERNOON.—MR. HIND.

Third and Fourth Classes.

1. Two weights sustain each other on two inclined planes having a common altitude, by means of a string parallel to the planes: compare the pressures.

2. Prove that

$$2(\sin^2 A \sin^2 B + \cos^2 A \cos^2 B) = 1 + \cos 2A \cos 2B.$$

3. PSp is any parameter of a parabola whose focus is S and latus rectum L, prove that

$$4\,SP \,.\, Sp = L(SP + Sp).$$

4. A parallelogram and a triangle upon the same base and between the same parallels revolve round the base as an axis: prove that the solid generated by the triangle equals one third of that generated by the parallelogram.

5. How much of the Earth's surface may be seen by a person raised n radii above it?

6. The ordinate to the axis of an ellipse is produced till it equals the corresponding subtangent: find the equation to the curve thus traced out, and its area.

7. Given the sum of $2n$ quantities in arithmetical progression and the sum of their squares, to find the quantities themselves.

8. Three equal weights are placed at the angles of an equilateral triangle without weight, which is suspended by an axis perpendicular to its plane bisecting one of its sides; find the centre of oscillation.

9. Prove *Newton's* fourth Lemma, (Sect. 1.) and by means of it, find the content of an oblate spheroid.

10. Differentiate

(1) $\log \dfrac{\sqrt{4ax + x^2} + x}{\sqrt{4ax + x^2} - x}$:

(2) $\dfrac{e^x \sqrt{e + x}}{e^{-x} \sqrt{e - x}}$:

and integrate

(1) $\dfrac{dx}{\sqrt{a + b\sqrt{x}}}$:

(2) $\dfrac{dx\sqrt{a^2 + x^2}}{x^8}$:

(3) $e^x \cos x\, dx$.

11. A vessel of given altitude empties itself through an orifice of given dimensions in its lowest point, and the upper surface descends with a given uniform velocity: find the content of the vessel.

12. A ray of light issuing from a point in the extreme ordinate of a parabola is incident in a direction parallel to the axis, and after two reflections at the curve meets the ordinate

again: prove that the length of the path described will be the same from whatever point in the ordinate the ray proceeds.

13. Find the area included between any two radii of a spiral where the angle contained between them is the measure of their ratio.

14. Given the velocity of projection equal to the velocity in a circle at the same distance, $\left(\text{force} \propto \frac{1}{D^2}\right)$; required the direction in which a body must be projected at a given distance, that the focus of the conic section described may bisect the semi-axis major, and determine the magnitudes and positions of the axes.

15. The plane of a vertical dial is inclined at an angle of 45° to the plane of the meridian in a latitude whose sine $= \frac{1}{\sqrt{3}}$: find the position of the substile, the altitude of the stile and the hour lines.

MONDAY EVENING.—Mr. Hind.

1. Shew that $y^{mn} - 1$ is divisible by either of the quantities $y^m - 1$ and $y^n - 1$ without a remainder.

2. In an isosceles plane triangle, prove, *geometrically*, that the versed sine of the vertical angle : radius :: the square of the base : twice the square of either side.

3. Given the m^{th} and n^{th} terms of an harmonical progression, to find the $(m+n)^{\text{th}}$ term.

4. The Earth being considered a perfect sphere, prove that at any place the length of a degree of latitude : the length of a degree of longitude :: radius : cosine of latitude.

5. Prove that $\cos(A+B)\sin(A-B) + \cos(B+C)\sin(B-C) + \cos(C+D)\sin(C-D) + \cos(D+A)\sin(D-A) = 0$.

Investigate the following formulæ;

$$\cos \theta = \frac{e^{\theta\sqrt{-1}} + e^{-\theta\sqrt{-1}}}{2}, \quad \sin \theta = \frac{e^{\theta\sqrt{-1}} - e^{-\theta\sqrt{-1}}}{2\sqrt{-1}};$$

and thence prove that

$$\frac{\sin A}{1 - \cos A} = \cot \frac{A}{2}.$$

✓

6. Find the time of emptying a sphere filled with fluid through an orifice in its lowest point.

7. Prove that

$$\log (n+1) = \log n + 2 \log \left(\frac{2n+2}{2n+1}\right) + \log \frac{(2n+1)^2}{(2n+1)^2 - 1}.$$

8. Find the surface of an equilateral and equi-angular spherical polygon of n sides, and determine the value of each of the angles when the surface equals half the surface of the sphere.

9. Required the radius of curvature of the curve whose equation is $\frac{x}{y} = \frac{a+y}{a-x}$, and determine the co-ordinates of the centre.

10. A ball whose elasticity : perfect elasticity :: $n:1$, is projected with a given velocity in a direction making an angle of 60° with the horizon, and when at its greatest height is reflected by a vertical plane: determine where the ball will again strike the horizon and the whole time of flight.

11. Investigate a differential expression for the surface of a solid of revolution; and apply it to find the surface of the solid generated by the figure of tangents revolving round its axis.

12. Shew how every case of oblique spherical triangles may be solved by *Napier's* rules *only*.

13. Investigate the apsidal equation, and shew what number of possible positive roots it can have, when the velocity is acquired from a finite distance and the force varies as D^{n-1}.

14. Given the logarithms of $1 + x$ and $1 + 2x$, shew how the logarithm of $1 + 3x$ may be computed.

15. Define the center of spontaneous rotation; shew generally how it may be found, and determine it when a straight rod of uniform density and given length is struck perpendicularly at a given point.

16. If the area of a curve between any two values of one abscissa can be expressed in finite terms, shew that the area between two values of any other abscissa of the same curve can be found.

17. The density of different parts of a circle varies as the square of the distance from the centre: find the velocity acquired by a corpuscle attracted towards this circle in a line passing through its centre and perpendicular to its plane, the attractive force of each particle varying as $\frac{1}{D^2}$.

18. State generally the principal of *Virtual Velocities:* and from it deduce the position of equilibrium of a straight rod of uniform density placed on two inclined planes.

19. Find the n^{th} differential of $\sqrt{1-x^2}$:

$$\text{find also} \int^{s} \frac{dx}{\sqrt{1-x^2}} \int \frac{dx}{\sqrt{1-x^2}};$$

and $\int dx\,(a^2 + b^2 - 2ab\cos x)^n$ from $x = 0$ to $x = 180°$; and determine the relation between x and y when

$$\frac{d^2 y}{dx^2} - A\frac{dy^2}{dx^2} + B\frac{dy^3}{dx^3} = 0.$$

20. Find the whole times of ascent and descent of a body urged by the force of gravity in a medium whereof the resistance varies as the square of the velocity, and give *Newton's* constructions.

21. Sum the following series:

$$(1) \quad \frac{2x}{1\,.\,3} + \frac{3x^2}{3\,.\,5} + \frac{4x^3}{5\,.\,7} + \&c. \textit{ in infinitum:}$$

(2) $\frac{\cos\theta}{1^2} - \frac{\cos 2\theta}{2^2} + \frac{\cos 3\theta}{3^2} -$ &c. *in infinitum:*

(3) $\tan\theta\left(\tan\frac{\theta}{2}\right)^2 + 2\tan\frac{\theta}{2}\left(\tan\frac{\theta}{4}\right)^2 + 4\tan\frac{\theta}{4}\left(\tan\frac{\theta}{8}\right)^2$

+ &c. to (n) terms.

22. Find the horary motion of the Moon's nodes in a circular orbit. *Newton*, Prop. 30. Book iii.

23. A ray of homogeneal light is refracted through a double convex lens: compare the densities in the different parts of the circle of chromatic dispersion.

24. The sum of the areas described by any number of bodies round a given point, multiplied by the respective masses of the bodies, is proportional to the time, if they be supposed to be acted upon *only* by their mutual attractions, and by the force tending to the given point.

TUESDAY MORNING.—Mr. Hind.

First and Second Classes.

1. A and B can do a piece of work in m days; B and C in n days: in what time can A and C do the same, it being supposed that A can do p times as much as B in a given time?

2. A normal drawn to a cissoid at the point where it cuts the generating circle, meets the axis produced in a certain point: prove that the line intercepted between this point and the vertex of the cissoid is divided into three equal parts by the centre and the further extremity of the diameter, of the generating circle.

3. The distance of the centre of gravity of a cycloid from the vertex $= \frac{7}{12}$ths of the axis: compare, from this, the contents of the solids generated by its revolution round the base and a tangent at the vertex.

4. Prove that the cube of any number and the number itself, being divided by 6, leave the same remainder.

5. Two straight rods equal in length are suspended by their extremities, one being of uniform density and the density of the other varying as the n^{th} power of the distance from the point of suspension; and they make small oscillations in times which are as $\sqrt{5} : \sqrt{6}$. Required the value of n.

6. An upright cylindrical vessel empties itself through an orifice in the base: compare the pressures upon the concave surface at first, and when half the time of emptying has elapsed.

7. Given the magnitude of a spherical surface, find the radius of the sphere so that the corresponding spherical segment may be the greatest possible.

8. Compare the momentum of a paraboloid with that of its inscribed cone having the same base and vertex, when they both revolve round their common axis.

9. Investigate the equation to the curve, in which the area has the same ratio to the square of the ordinate, that the ordinate has to the abscissa.

10. Given the latitudes and longitudes of two places on the Earth's surface, to find their distance.

11. Find the integral of

$$x^m\, dx\, (\log x)^m \text{ from } x = 0 \text{ to } x = 1.$$

12. A string wrapped round a cylindrical annulus of uniform density whose radii are R and r, passes over a fixed pulley, and has a weight attached to it: find the space descended by the annulus in a given time.

13. Given the time, construct for the inclination of the lunar orbit to the plane of the ecliptic.

Newton, Prop. 35. Book iii.

TUESDAY AFTERNOON.—Mr. Hind.

Fifth and Sixth Classes.

1. Find the present worth of *P£* due n years hence, at r per cent. discount.

2. Investigate the rule for the extraction of the square root in whole numbers; and determine, *generally*, the limit which the remainder after any operation cannot exceed.

3. Shew that the content of a sphere : the content of the greatest cone that can be inscribed in it :: $3^3 : 2^3$.

4. Given the ratios of the sines of the angles of a plane triangle, and the radius of the inscribed circle; to construct the triangle.

5. The orifices in the equal bases of two upright prismatic vessels are in the ratio of $2 : 1$, and the vessels are emptied in equal times; compare their altitudes.

6. Prove that *Waring's* solution of a biquadratic equation fails when all the roots are impossible, and of the form

$$a \pm b\sqrt{-1}.$$

7. Determine the point in *P's* orbit, (Sect. 11,) where the tangential ablatitious force is a mean proportional between the addititious and central ablatitious forces.

8. The roots of the equation $6x^4 - 43x^3 + 107x^2 - 108x + 36 = 0$, are of the form a, b, $\frac{a}{b}$ and $\frac{b}{a}$, find them: and shew what relation exists between the co-efficients of a cubic whose roots are of the form $+a$, $-a$ and $\pm b$.

9. Find the point at which a ray of light parallel to the axis must be incident upon a concave spherical reflector, that after two reflections it may cut the axis in a given angle.

10. The equation to a curve is $y = x^3 - 9x^2 + 24x + 16$; determine the values of the abscissa when the ordinate is a maximum, and when a minimum; and find the area included between those ordinates.

11. Differentiate

(1) $\log(2x + 1 + 2\sqrt{1 + x + x^2})$:

(2) $\cos\theta + \sec\theta$.

Integrate

(1) $\dfrac{x^2\,dx}{(1 + x^4)^2}$: (2) $\dfrac{x^{\frac{1}{2}}\,dx}{\sqrt{a + b\sqrt{x}}}$: (3) $a\,dx\int b\,dx\int c\,dx$:

and find the values of

(1) $\dfrac{a^{\log x} - 1}{\log x}$ when $x = 1$: (2) $\dfrac{\log\tan x}{\log\tan\frac{x}{2}}$ when $x = 0$.

12. The magnitudes of three perfectly elastic bodies are in harmonical progression: prove that the momentum communicated to either of the extremes by the impact of the other equals the momentum of the mean moving with the velocity of the impinging body before impact.

13. Given the latitude of the place and the length of the day, to find the time of the year.

14. If a body describe the arc of a cycloid by a force acting parallel to its base; prove that the force varies inversely as $2\sin\theta - \sin 2\theta$; θ being the corresponding arc of the generating circle reckoned from the vertex.

TUESDAY AFTERNOON.—Mr. Chevallier.

Third and Fourth Classes.

1. $S_1, S_2, S_3 \ldots\ldots S_n$ being the sums of n geometric series continued *in infinitum*, the first term of which is 1, and the

common ratio $\frac{1}{r}, \frac{1}{r^2}, \frac{1}{r^3}, \ldots \frac{1}{r^n}$, respectively. Required the sum of their reciprocals,

$$\frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} \cdots + \frac{1}{S_n}.$$

2. Shew that cones and cylinders upon equal bases are to one another as their altitudes.

3. P supports W upon an axle, by means of a perpetual screw acting upon the circumference of the wheel. Required their proportion.

4. A sphere of glass and another of water being placed in air, what must be the proportion of their radii, that their magnifying powers may be the same?

5. A life-boat contains 100 cubic feet of wood, specific gravity .8; and 50 feet of air, specific gravity .0012. When filled with fresh water, what weight of iron ballast, specific gravity 7.645, must be thrown in before it will begin to sink?

6. Explain the different uses of a fly-wheel in machinery.

7. The velocity in an ellipse at the greatest distance is half that with which a body would move in a parabola at the same distance. What is the eccentricity of the ellipse?

8. Shew that the stereographic projection of a great circle of a sphere is a circle; and find the radius.

9. P descends vertically, drawing Q over a fixed pulley. Find the pressure upon the axis of the pulley; and its value when P is indefinitely increased.

10. A pendulum is composed of two thin wires of equal length, at right angles to each other at the point of suspension, and vibrating in their own plane. Find the time of a small oscillation; and the angle at which they must be inclined to each other, so that the time of oscillation may be doubled.

11. What effects are produced by aberration in the apparent places of the Moon and the Planets?

12. Shew that the inclination of the Moon's orbit is the greatest, when the line of the nodes is in syzygy; and the least, when the nodes are in quadrature and the Moon in syzygy.

TUESDAY EVENING.—Mr. Chevallier.

1. Prove the rule for single position: state to what limitations it is subject; and apply it to find such a number that when divided by 3, 4 and 5, respectively, the sum of the quotients may be 94.

2. Shew that if an equation have two equal roots, and the terms are multiplied by the terms of an arithmetic progression, the result will $= 0$.

3. Find a quantity, which when multiplied into $a^{\frac{1}{3}} - b^{\frac{1}{3}}$ renders the product rational.

4. Given the place of a planet at noon, on March 20th, in Libra $3^\circ.4'.30''$; on the 21st in $8^\circ.7'.7''$; on the 22d in $13^\circ.19'.30''$; and on the 23d in $18^\circ.41'.44''$; to find its place on the 22d at 6^h.

5. Sum the series

$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \text{\&c., to } n \text{ terms and } \textit{in infinitum}.$$

$$1^2 + 3^2 + 5^2 + \text{\&c., to } n \text{ terms}.$$

$$1 + 2x + 11x^2 + 43x^3 + \text{\&c., } \textit{in infinitum}.$$

6. If Jupiter and Saturn are in conjunction with one another, and in opposition to the Sun, on a given day; and their periodic times are 12 years and 29.5 years respectively; when will they again be in the same position?

7. A weight W is raised upon a moveable pulley. The two extremities of the cord are wound in different directions about two cylinders, which have a common axis but different

radii; and the power P descends unwinding a string from a wheel of given radius upon the same axis. What is the force which accelerates P's descent, when the strings are parallel to each other?

✓ 8. In the scale of Reaumur's thermometer, the freezing point of water is 0, and the boiling point 80°. In the centigrade thermometer, those points are 0 and 100° respectively. What will be the degree of heat marked by each, when Fahrenheit's thermometer stands at 59°?

9. Given the altitude of the Sun, and the breadth of the penumbra which the top of a mountain throws upon a horizontal plane, to find the height of the mountain.

10. Explain Atwood's machine; and mention some of the facts which it establishes in the theory of motion uniformly accelerated or retarded.

11. A plane of given form and area is supported in the air as a kite, the wind acting in a direction parallel to the horizon: the weight of the string and materials being (w), and the horizontal pressure of the wind equivalent to a weight (p) upon each square foot, required the angle made by the plane with the horizon; and the greatest weight which it can support.

12. When the force by which a watch-balance is actuated varies as the n^{th} power of the distance from the point of the spiral spring's quiescence, find the alteration in the daily rate, in consequence of a given change in the arc of vibration.

13. Explain the method of deducing the Sun's parallax from the transit of an inferior planet.

14. A seconds' pendulum of given length, in the form of a thin rectangular bar, suspended at the middle of its extremity by an axis perpendicular to its plane, is carried to the top of a mountain. The length of the bar is diminished by a given quantity in consequence of a change of temperature, the breadth remaining the same, and it loses t'' in a day. What is the height of the mountain?

15. A cannon ball weighing 24lbs. strikes a wall with a velocity of 1700 feet. Find the weight of a beam, terminated by a hemisphere of the same diameter as that of the ball, which, when moved with a velocity of 10 feet, may penetrate to the same depth; and the weight of a similar beam, which may have the same effect in *shaking* the wall.

16. A body not affected by gravity falls down the axis of a thin cylindrical tube infinite in length, the particles of which attract with a force which varies inversely as the square of the distance. Find the velocity acquired in falling through a given space.

17. Four persons A, B, C, D, in order, cut a pack of cards, replacing them after each cut, on condition that the first who cuts a heart shall win. What are their respective probabilities of success?

18. A ship (P) begins to sail towards another (B) from a fixed point (C). At the same instant B begins to move in a direction perpendicular to P's first motion: P is always found in the line joining C and B; but can only accelerate her rate of sailing so as to retain the same distance from B as at first. What is the curve traced by P?

19. Integrate

$$\frac{dx}{\sqrt{x^2+y^2}} - \frac{x\,dy}{y\sqrt{x^2+y^2}} = 0;$$

$$dy + y\,dx = a x^3 dx;$$

$$dx\,dy - (x+a)\,d^2 y - \frac{x\,dy^2}{b} = 0.$$

20. Two material points S and P, the mass of the first being twice that of the second, attract each other with a force which varies inversely as the square of the distance. When they have approached each other by half their original distance, P receives a new perpendicular impulse, which communicates to it a velocity equal to that which S has acquired. What curve is now described by each about the other?

21. Find the whole variation in the inclination of the Moon's orbit, as the Moon moves from quadrature to syzygy; the line of the nodes lying in quadrature.

Newt. Lib. iii. Prop. 34. Cor. 4.

22. Shew that, if a body oscillates in a cycloid, in a medium the resistance of which is constant, the successive altitudes to which it will rise are in arithmetical progression.

23. The axis of Archimedes' screw is inclined at a given angle to the horizontal section of the water. Find the highest and lowest points of the spiral tube, its point of inflection, and the quantity of water which can be raised in a given time by a given power.

24. If a body is acted upon by any forces which would, if separately communicated, cause it to revolve about given axes with given angular velocities; find its axis of rotation, and its angular velocity: and apply the conclusion where there are 3 axes at right angles to each other.

1823.

MONDAY MORNING.—Mr. Hind.

First and Second Classes.

1. A given annuity which is to continue $3n$ years, is left equally between A and B: A receives the whole for n years, and B the whole for the remainder of the time; it is required to find the present worth of the annuity, and the rate of compound interest.

2. Find the values of θ which satisfy the equation

$$2 \sin^2 3\theta + \sin^2 6\theta = 2, \quad \text{radius being} = 1.$$

3. Two bodies A and B descend from the same extremity of the vertical diameter of a circle, one down the diameter, and the other down the chord of 30°. Find the ratio of A to B, when their centre of gravity moves along the chord of 120°.

4. Given the sum of three quantities in geometrical progression, and the sum of their reciprocals, to find the quantities themselves.

5. The angles of a plane triangle are A, B, C; it is required to prove that the perimeter of the triangle : the diameter of its inscribed circle :: $\text{rad}^3 : \tan\frac{A}{2} \tan\frac{B}{2} \tan\frac{C}{2}$.

6. In an equation of n dimensions, the second and third terms may be taken away by the same transformation when the square of the sum of the roots : the sum of their squares :: $n : 1$. Required a proof.

7. Three points being given in position; it is required to draw a straight line through one of them, so that the rectangle of the perpendiculars let fall upon it from the other two may be the greatest possible.

8. A ray of light issues from the extremity of the diameter of a semi-circle, and is reflected by the circumference: determine the point of incidence, so that after reflection, the ray may pass through a given point in the diameter produced.

9. A semi-circular area is placed with its vertex upon a horizontal plane; find the time of one of its small oscillations.

10. A body projected in a direction parallel to the horizon, and acted upon by the force of gravity, describes a common cycloid; shew that the resistance of the medium, and the velocity at any point, vary respectively as $\sin\frac{\theta}{2}$, and $\cos\frac{\theta}{2}$, θ being the corresponding arc of the generating circle.

11. A chain of uniform density is suspended at its extremities by means of two tacks in the same horizontal line at a given distance from each other; find the length of the chain so that the stress upon either tack may be equal to the chain's weight.

12. Integrate $\dfrac{dx}{\sqrt[m]{1 - x^m}}$.

13. Find the horary variation of the inclination of the lunar orbit to the plane of the ecliptic. *Newton*, Prop. 34. Book 3.

14. Solve the functional equation

$$(\psi a^x)^2 \psi\left(\frac{1 - a^x}{1 + a^x}\right) = b^2 a^x.$$

MONDAY AFTERNOON.—Mr. Hind.

Fifth and Sixth Classes.

1. In what time will the amount of P£. at r per cent. simple interest be equal to p times the interest of the same

sum, and what is the rate per cent. when the required time is q years?

2. Prove *geometrically* that

$$\text{versin } A : \sin A :: \tan \frac{A}{2} : \text{rad.}$$

3. Of the two quantities $a^6 + a^4 b^2 + a^2 b^4 + b^6$ and $(a^3 + b^3)^2$, shew which is the greater.

4. Divide the length of a given inclined plane into three parts so that the times of descent down them may be equal.

5. In any plane triangle, prove that the sines of the angles are inversely as the perpendiculars let fall from them upon the opposite sides.

6. Two bodies, A and B, whose elasticity is m, moving in opposite directions with velocities a and b impinge directly upon each other: find the distance between them when t'' from the moment of impact have elapsed.

7. The roots of the equation $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$ are of the form $a+1$, $a-1$, $b+1$, $b-1$: find them.

8. Where must a ray of light parallel to the axis of a concave spherical reflector be incident, that after reflection it may divide the radius in the ratio of $\sqrt{3} - 1 : 1$?

9. The sum of a series of quantities in geometrical progression wanting the first term, is equal to the sum of all the terms except the last, multiplied by the common ratio. Required a proof.

10. Find the angular distance of a body from the vertex of a common parabola where the velocity is equal to half the greatest velocity.

11. Determine *geometrically* that point in the hypothenuse of a given right-angled triangle whose base is parallel to the horizon, from which the time of a body's descent to the right angle may be the least possible.

12. Shew that *Cardan's* rule for the solution of a cubic equation is applicable when all the roots are possible and two of

them equal; and by means of it find the roots of the equation $x^3 + 6x^2 - 32 = 0$.

13. Find the differential of an arc the tangent of whose half is x; and integrate

$$\frac{x\,dx}{\sqrt{a^4 x^{-4} + 1}},\ \frac{x^{\frac{1}{2}}\,dx}{a^3 - 5x^3} \text{ and } \frac{d\theta}{(\tan\theta)^2}.$$

14. Sum the series:

$$\frac{1}{m} + \frac{2}{m(m+a)} + \frac{2(a+2)}{m(m+a)(m+2a)} + \&c. \textit{ in infinitum},$$

when m is greater than 2.

15. Divide the arc of a cycloid into two parts so that the times of a body's oscillating through them may be in the ratio of 1 : 5.

16. Two places in the same latitude whose difference of longitude is l, are distant a miles from each other: find their latitude.

17. The difference of the forces on P and p (*Newt.* Sect. 9.) $\propto \frac{1}{CP^3}$: required a proof.

18. A given paraboloid filled with fluid is placed with its vertex downwards and its axis vertical; determine the time of emptying one half of its content through a given orifice in the vertex.

MONDAY AFTERNOON.—MR. HIGMAN.

Third and Fourth Classes.

1. Transform the equation $x^3 - px^2 + qx - r = 0$ whose roots are a, b, c, into one whose roots are

$$\left(\frac{a}{b} + \frac{b}{a}\right),\ \left(\frac{a}{c} + \frac{c}{a}\right),\ \left(\frac{b}{c} + \frac{c}{b}\right).$$

2. Find the number of different triangles into which a polygon of n sides may be divided by lines joining the angular points.

3. If the velocities of two balls A and B, whose elasticity is e, be a and b before impact, and u and v after; also if α and β be the velocities lost and gained, then will

$$A\,a^2 + B\,b^2 = A\,u^2 + B\,v^2 + \frac{1-e}{1+e}\left\{A\,\alpha^2 + B\,\beta^2\right\}.$$

4. Find the attraction of a rectangle on a corpuscle situated in one of its sides produced, in a direction perpendicular to the other side: the force tending to each particle of the rectangle varying inversely as the square of the distance.

5. If there be taken the evolute of a logarithmic spiral, the evolute of that evolute, and so on *ad infinitum*, find the sum of the arcs of all the successive evolutes.

6. Sum the series:

(1) $\dfrac{1}{3.5.7} + \dfrac{2}{4.6.8} + \dfrac{3}{5.7.9} + \&c.$ to n terms.

(2) $\dfrac{1}{1.4.7} + \dfrac{1}{2.6.9} + \dfrac{1}{3.8.11} + \&c.$ *in infinitum.*

(3) $1 + n\cos\theta + \dfrac{n(n-1)}{1.2}\cos 2\theta$

$$+ \frac{n(n-1)(n-2)}{1.2.3}\cos 3\theta + \&c.$$

7. If a ray of light refracted into a sphere, emerge from it after any given number of reflections; determine the distance of the incident ray from the axis, when the arc of the circle intercepted between the axis and the point of emergence is a minimum.

8. Find the center of resistance of a semi-circle revolving round its diameter in a medium, whose resistance $\propto$ (velocity)^2.

9. If z = true zenith distance of a planet, p = its parallax at that distance, and P = horizontal parallax, then

$$\tan\left(\frac{z}{2}+p\right)=\tan\frac{z}{2}\tan^2\left(45^\circ+\frac{P}{2}\right).$$

10. If the force $\propto \frac{1}{\text{dist.}^4}$, and a body be projected at an apse with the velocity acquired in descending from an infinite distance to that point, construct the curve described, and find the time of descent to the centre.

11. A body descends down the convex side of a logarithmic curve placed with its asymptote parallel to the horizon, find where it leaves the curve.

12. If S be the momentum of inertia of a system, in respect of an axis which passes through its centre of gravity, S' the momentum of inertia of the same system, in respect of an axis parallel to the first and distant from it by the space k, and M the mass of the system, then

$$S' = S + Mk^2.$$

MONDAY EVENING.—Mr. Higman.

1. Prove that $(Aa + Bb + Cc + \ldots\ldots)^2 = (A + B + C + \ldots\ldots)(Aa^2 + Bb^2 + Cc^2 + \ldots\ldots) - AB(a-b)^2 - AC(a-c)^2 - BC(b-c)^2 - \ldots.$

2. If through any point O within a triangle, three straight lines be drawn from the angles A, B, C meeting the opposite sides in a, b, c, then will $\frac{Oa}{Aa}+\frac{Ob}{Bb}+\frac{Oc}{Cc}=1.$

3. If from any point in a rectangular hyperbola whose axis is vertical, two lines be drawn to the extremities of the axis major, the times of descent down them will be equal.

4. Prove that the periodic time of a body revolving in an ellipse round the focus $= \frac{2\pi a^{\frac{3}{2}}}{\sqrt{m}}$; where a = semi-axis, and

$m =$ force at a distance 1; and apply this result to deduce the actual time of falling down AC in Prop. 32. Sect. 7.

5. The sine of half the angle that measures the duration of the shortest twilight $= \dfrac{\sin 9^\circ}{\cos \text{lat}}$.

6. Find the equation of the caustic of the parabola, when the rays are incident perpendicular to the axis. Trace the caustic, and find the angles at which it cuts the axis, its maximum ordinate, its area, and its length.

7. If a be a root of Des Cartes's reducing cubic, then will the four roots of the equation $x^4 + q x^2 + r x + s = 0$, be

$$-\frac{\sqrt{a}}{2} \pm \sqrt{\left(-\frac{q}{2} - \frac{a}{4} + \frac{r}{2\sqrt{a}}\right)},$$

$$\text{and} + \frac{\sqrt{a}}{2} \pm \sqrt{\left(-\frac{q}{2} - \frac{a}{4} - \frac{r}{2\sqrt{a}}\right)}.$$

8. Two cylindrical vessels of given dimensions, containing given quantities of water, are made to communicate with each other by means of a small orifice at their bases; find the time elapsed before the water stands at the same altitude in both vessels.

9. If a comet move in a hyperbola whose semi-axis $= a$, and eccentricity $= a e$, its place at the end of t'' after leaving the perihelion may be determined from the equations:

$$(1) \quad t = \frac{P a^{\frac{3}{2}}}{2\pi} \left\{ e \tan \theta - \log \tan \left(\frac{\pi}{4} + \frac{\theta}{2} \right) \right\}.$$

$$(2) \quad \tan \frac{v}{2} = \sqrt{\left(\frac{e+1}{e-1}\right)} \tan \frac{\theta}{2}.$$

where $v =$ true anomaly reckoned from the perihelion, and $P =$ Earth's periodic time, her mean distance being 1.

10. Shew that in consequence of the mean disturbing force of the Sun in the direction of the radius vector, the distance of

the Moon from the Earth is increased by a 358th part, and her angular velocity diminished by a 179th part.

11. Integrate

(1) $$\frac{x^4\,dx}{\sqrt{(2ax - x^2)}}.$$

(2) $$\frac{dx}{\sqrt{(1 - x^2)}}.\log x \text{ between } x = 0, \text{ and } x = 1.$$

(3) $$a y\, dy - b y^2\, dx + c x\, dx = 0.$$

12. If AB be an elliptic quadrant, CP, CD semi-conjugate diameters, PF perpendicular to CD, K the point, in which CD produced meets the circumscribing circle, and KMQ a line perpendicular to the major axis meeting the ellipse in Q, then will arc BP − arc $AQ = CF$.

13. The square of the area of any one of the faces of a triangular pyramid, is equal to the sum of the squares of the other three, minus twice the rectangle contained by the product of every two and the cosine of their inclination.

14. Sum the following series:

(1) $$\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \ldots \textit{ad inf.} \text{ when } n \text{ is infinite.}$$

(2) $$\frac{x^2}{1.2} + \frac{x^5}{1.2.3.4.5} + \frac{x^8}{1.2.3.4.5.6.7.8} + \ldots \textit{ad inf.}$$

(3) $$\frac{1}{2.1^2} + \frac{1}{2^2.2^2} + \frac{1}{2^3.3^2} + \ldots \textit{ad infinitum.}$$

15. Amongst all the axes passing through the centre of gravity of a triangle in its own plane, find that for which the momentum of inertia is a maximum or a minimum.

16. If a ray of light $QACS$ be refracted through a prism IKL in a plane perpendicular to its axis, and if the vertical

angle $KIL = \alpha$, $QAK = \theta$, $ACL = \phi$, and the whole deviation of the ray $= \delta$, then will

$$\tan\left(\phi - \frac{\alpha}{2}\right) = \frac{\tan\left(\theta + \frac{\delta + \alpha}{2}\right)\tan\frac{\delta + \alpha}{2}}{\tan\frac{\alpha}{2}}.$$

17. Find the different positions of equilibrium of a parabola floating in a fluid with its vertex immersed.

18. If the longitudes of a planet in three different points of its orbit be denoted by a, b, c, and its latitudes at those points by α, β, γ; then will $\tan\beta \,.\, \sin(c - a) = \tan\alpha \,.\, \sin(c - b) + \tan\gamma \,.\, \sin(c - a)$.

19. If x be a very large number, and e the base of Napier's system, then will

$$1 \,.\, 2 \,.\, 3 \,.\, 4 \ldots\ldots x = \left(\frac{x}{e}\right)^x \sqrt{(2\pi x)} \text{ nearly.}$$

20. A and B are at play together, and the latter having lost p stakes, is determined to play till he has won them again; find the probability that this never takes place, supposing the play to continue without limitation; his number of chances b for winning any assigned game being less than a that for the contrary.

21. If the force $\propto \frac{1}{\text{dist.}}$, then the time of descent to the centre $= a\sqrt{\frac{\pi}{2m}}$, where $a =$ whole distance, and $m =$ force at a distance 1 from the centre.

22. Required the curve, which within its own arc, its evolute and radius of curvature shall contain the least area.

23. When pulses are propagated through an elastic medium, the several parts going and returning by a very short reciprocal motion; they are accelerated and retarded according to the law of a pendulum oscillating in a cycloid.

Newton, Lib. 2. Prop. 47.

24. If two bodies be projected at equal angles of elevation, and with equal velocities, one in a non-resisting medium, and the other in a medium whose resistance corresponding to the velocity $v = n v^2$; also, if s' be the arc of the parabola, and s the arc of the other curve described by the bodies when they are moving in directions making equal angles with the horizon, then will $e^{2ns} = 1 + 2ns'$.

TUESDAY MORNING.—Mr. Higman.

First and Second Classes.

1. Find the greatest term of the expansion of $(a + b)^n$.

2. The product of all the lines that can be drawn from one of the angles of a regular polygon of n sides, inscribed in a circle whose radius is a, to all the other angular points $= n a^{n-1}$.

3. If the sun's longitude $= c$, and the obliquity of the ecliptic $= \phi$, then will the equation of time arising from the obliquity of the ecliptic

$$= \tan^2 \frac{\phi}{2} \sin 2c - \frac{1}{2} \tan^4 \frac{\phi}{2} \sin 4c + \frac{1}{3} \tan^6 \frac{\phi}{2} \sin 6c - \&c.$$

ad infinitum converted into time.

4. A beam of given length and weight is placed with one end on a vertical, and the other on a horizontal plane: find the force necessary to keep it at rest, and the pressures on the two planes.

5. Find the perihelion distance of the comet, moving in the plane of the ecliptic, that stays the longest time within the earth's orbit.

6. If P be any rational function of x, in which the highest power of x is less than n; and if $A, B, C, D, \ldots K$ be the values

$$\text{of } P, \frac{dP}{dx}, \frac{d^2P}{dx^2}, \frac{d^3P}{dx^3} \cdots\cdots \frac{d^{n-1}P}{dx^{n-1}}$$

when $x = a$, then

$$\frac{P}{(x-a)^n} = \frac{A}{(x-a)^n} + \frac{B}{(x-a)^{n-1}} + \frac{C}{1.2.(x-a)^{n-2}}$$

$$+ \frac{D}{1.2.3.(x-a)^{n-3}} + \dots + \frac{K}{1.2\dots(n-1)(x-a)}.$$

7. Explain fully the motion of the apsides, and the variation of the eccentricity of *P's* orbit, Cor^r. 8 and 9, Prop. 66.

8. A sphere of given weight and dimensions, descends in a fluid; find the pressure on the bottom of the vessel that contains it, arising from the action of the sphere on the fluid.

9. If f, f', f'' ...... be the focal lengths of any number of contiguous lenses, the thickness of each being very small, and F be the focal length of the compound lens, then

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f'} + \frac{1}{f''} + \dots\dots$$

10. If in the spherical triangle ABC, c and C be constant, and the other angles and sides variable, then will AC and BC be the corresponding values of ϕ and θ in the differential equation

$$\frac{d\phi}{\sqrt{(1-e^2\sin^2\phi)}} + \frac{d\theta}{\sqrt{(1-e^2\sin^2\theta)}} = 0 \text{ where } e = \frac{\sin C}{\sin c}.$$

11. Putting A and B for the sectors CAP, CAp of a rectangular hyperbola, whose semi-axis $CA = 1$, and calling the abscissas CN, Cn, the *cosines*, and the ordinates PN, pn *the sines* of A and B, then will

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B,$$

$$\text{and } \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B.$$

12. If a body describing a spiral in a medium whose density $\propto \frac{1}{\text{dist.}}$, cut the radius vector at B in the same angle as at A, and with a velocity, which is to that at $A :: \sqrt{SA} : \sqrt{SB}$, then

will the distances at which the body cuts the radius vector, and the times of successive revolutions be in geometric progression. *Newton,* Cor. 7. Prop. 15. Lib. 2.

TUESDAY AFTERNOON.—Mr. Higman.

Fifth and Sixth Classes.

1. In every geometrical progression consisting of an odd number of terms, the sum of the squares of the terms is equal to the sum of all the terms multiplied by the excess of the odd terms above the even.

2. Take away the third term of the equation

$$x^3 - 6x^2 + 9x - 20 = 0.$$

3. On a lever of uniform density, every inch weighing w oz. a weight of W oz. is suspended at a given distance from the fulcrum which is placed at one extremity. What must be the length of the lever, so that the whole may be supported by the least possible power acting in an opposite direction at the other extremity?

4. The initial force accelerating a body down a circular arc is to the force accelerating it down the chord as 2 to 1 ultimately.

5. If $S =$ surface of a portion of the earth $ABCD$, AB being an arc of the equator, and AC, BD two arcs of circles of latitude; also if $AB = c$, $AC = a$, $BD = b$, then will

$$\tan \frac{S}{2} = \frac{\sin \frac{a+b}{2}}{\cos \frac{a-b}{2}} \tan \frac{c}{2}.$$

6. Determine that point of the cubical parabola where the curvature is the greatest.

7. If in an ellipse there be taken three abscissas in arithmetic progression, the radius vectors drawn from the focus to

the extremities of the ordinates at those points will also be in arithmetic progression.

8. Find the sum of $1 + \frac{\alpha}{2} + \frac{\beta}{3} + \frac{\gamma}{4} + \&c.$ α, β, γ, &c. being the co-efficients of the expansion of $(a + b)^n$.

9. Compare the resistances on a globe and the circumscribing cylinder moving in a fluid with equal velocities in the direction of the axis of the cylinder.

10. A given weight is suspended by a string and immersed in a river whose inclination to the horizon is known. Having observed the angle which the string makes with a vertical line, it is required to determine the velocity of the stream.

11. Sum the series:

(1) $1 + 3x + 5x^2 + 7x^3 + \&c.$ *ad infinitum.*

(2) $\frac{1}{1.2.4} + \frac{1}{2.3.5} + \frac{1}{3.4.6} + \&c.$ to n terms by increments.

12. Find the integrals of $\frac{x\,dx}{\sqrt{(a^4 + x^4)}}$; and of $v^2\,dx$ where $v = \log x$.

13. If the focus of incident rays be placed any where in the axis of an elliptic reflector, find the geometrical focus of reflected rays.

14. If A, A' denote the intensities of two lights, and n, n' the number of laminæ of any substance, through which when the lights are viewed they appear equally obscure; also the light lost in passing through each lamina being supposed to be $\frac{1}{m^{\text{th}}}$ of that which entered it; then will

$$A : A' :: \left(\frac{m-1}{m}\right)^{n'} : \left(\frac{m-1}{m}\right)^{n}.$$

15. In consequence of the aberration of light, every star appears to describe an ellipse in the heavens, of which the true place of the star is the center. Prove this, and find the axes of the ellipse.

16. If the force vary inversely as the square of the distance, and $m =$ force at a distance 1, and $l =$ latus rectum of the conic section, then the area described in $1'' = \sqrt{\frac{l\,m}{8}}$.

17. Trace the curve whose equation is $y = \sec x$, draw a tangent to it, and find its area.

18. Find the value of Q in *Newton*, Prop. 41.

TUESDAY AFTERNOON.—Mr. Hind.

Third and Fourth Classes.

1. Extract the square root of $\cos 4A \pm \sqrt{-1}\,\sin 4A$.

2. If tangents drawn to any two points of an ellipse meet each other; shew that their lengths are inversely as the sines of the angles which they make with the lines drawn to either focus.

3. There are four numbers, the first three of which are in arithmetical, and the last three in harmonical progression; it is required to prove that the first has to the second the same ratio which the third has to the fourth.

4. If $2S$ be equal to the sum of the sides of a plane triangle, A, B the angles opposite to the sides a, b, respectively, then is

$$\sin^2\frac{A}{2} : \sin^2\frac{B}{2} :: \frac{a}{b} : \frac{S-a}{S-b}.$$ Required a proof.

5. The beam of a false balance being of uniform density and thickness, it is required to shew that the lengths of the arms are respectively proportional to the differences between the true and apparent weights.

6. In the equation $x^3 - p\,x^2 + q\,x - r = 0$, prove that the sum of the products of the roots and their reciprocals taken three and three together : product of the roots :: $1 + p^2 + q^2 + r^2 : r^2$.

7. In any right-angled plane triangle, prove that twice the side of the inscribed square is an harmonical mean between the sides containing the right angle.

8. Shew that $\log \sec \theta = \frac{\tan^2 \theta}{2} - \frac{\tan^4 \theta}{4} + \frac{\tan^6 \theta}{6} - \&c.$ and thence deduce the sum of the series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \&c. \textit{ in infinitum.}$$

9. Determine the angular distance of a body from the vertex of an ellipse whose eccentricity $= \frac{1}{2}$, at which the velocity : greatest velocity :: $1 : \sqrt{3}$.

10. A cylinder of given length is just immersed vertically in two fluids whose specific gravities are as $1 : 2$, and the pressures of the fluids upon the convex surface are as $2 : 3$; find the length of the cylinder immersed in each fluid.

11. There are three plane reflectors, two of which are at right angles to each other, and a ray of light is incident upon the third, and reflected successively by each of them; it is required to shew that the angle between the first incident and last reflected rays is equal to twice the angle of incidence upon the first surface.

12. Determine the length of a straight line drawn through the centre of gravity of a given isosceles triangle, making a given angle with the base, and terminated by the sides.

13. AN and NP are the abscissa and ordinate of a cissoid, the diameter of whose generating circle is AB: AP is joined and NQ always taken equal to it. Prove that the whole area of the curve traced out by $Q : AB^2 :: 4 : 3$.

14. Given the latitude of a place, find the time of the year when a given star rises at a given hour.

TUESDAY EVENING.—Mr. Hind.

1. Extract the fourth root of $m^2(m^2-3n^2)+n^2(n^2-3m^2)$ $+ 4(m-n)(m+n)mn\sqrt{-1}$.

2. Prove the following Formulæ:

$$\tan^2\frac{A}{2}=\frac{2\sin A-\sin 2A}{2\sin A+\sin 2A} \text{ and}$$

$$\tan nA=\frac{\sin A+\sin 3A+\&c.\text{ to } n \text{ terms}}{\cos A+\cos 3A+\&c.\text{ to } n \text{ terms}}\text{; and}$$

from the former, deduce the tangent of 15°.

3. Find three fractions having different prime denominators, whose sum shall be $1\frac{23}{30}$.

4. If three roots of an equation be nearly equal to one another, and much less than all the others, shew that an approximation may be made to them by the solution of a cubic.

5. Every prism having a triangular base, may be divided into three pyramids, that have triangular bases, and are equal to one another. *Euclid*, Prop. 7. Book 12.

6. Explain the construction, adjustment and use of the zenith sector.

7. Find the greatest triangle that can be inscribed in a given circle.

8. Two given spheres are situated at the extremities of the diameter of a given circle: determine the position of an eye in the circumference, where the surface seen is the greatest possible.

9. A triangle is described about an ellipse, prove that the products of the alternate segments of the sides, made by the points of contact, are equal.

10. Prove that $\sin A+\sin 2A+\sin 3A+\&c.$ is a recurring series; find the scale of relation, and by means of it the sum of n terms.

11. P descending vertically draws Q up an inclined plane by means of a string passing over a pulley above it: find the velocity acquired by P in describing a given space.

12. Shew that the velocity acquired by a body in falling from infinity to the earth's centre, is to the velocity of a secondary at the earth's surface :: $\sqrt{3} : 1$.

13. Find where an inferior planet will appear stationary, supposing the force of gravity to vary inversely as the cube of the distance, and the orbits of the earth and planet to be circular.

14. In *Cotes's* first spiral, it is required to shew that the successive distances at which the curve cuts the apsidal line may be represented by a series of the form,

$$\frac{c}{c^2+1}, \quad \frac{c^2}{c^4+1}, \quad \frac{c^3}{c^6+1}, \ldots \frac{c^n}{c^{2n}+1}.$$

15. Determine the pressure upon the axis of any vibrating body in any given position.

16. Investigate the nature of the surface, such that if two lights of given intensities be placed in two given points, every point in it may be proportionally illuminated by each.

17. Integrate

$$\frac{d\,x}{(a + b \sin x)^2} \text{ and } \frac{x^m\,d\,x}{(x + \sqrt{1 + x^2})^4};$$

and find the relation of x and y in the equation,

$$\frac{d^2 y}{d x^2} + 2x \frac{d y}{d x} + (y - a) x^2 = 0.$$

18. Find the equation to a curve surface in which the normals from every point meet a given plane in a given straight line.

19. A given cylinder filled with fluid revolves round its vertical axis with a given uniform velocity: find what quantity of fluid will escape if a small orifice be made in the centre of its base.

20. Determine the conditions of equilibrium of a material point situated in a canal of indefinitely small dimensions and acted upon by any number of forces.

21. If the force of gravity be uniform and act perpendicularly to the plane of the horizon; it is required to determine the motion of a projectile in a medium whose resistance is proportional to the velocity. *Newton*, Prop. 4. Book 2.

22. If a prism be turned round its axis which is perpendicular to the incident rays of the sun, till the spectrum neither ascend nor descend, shew that the refractions at the points of incidence and emergence are equal.

23. To determine the mean motion of the Moon's nodes. *Newton*, Prop. 32. Book 3.

24. If three planes be at right angles to each other, find the equation to the surface to which if a tangent plane be drawn, the content of the solid formed by this and the other three planes will be constant.

1824.

MONDAY MORNING.—Mr. Hughes.

First and Second Classes.

1. The area of a floor is 403ft. 7in. and length 27ft. 10in. find the width by duodecimals.

2. Find the value of A in the equation

$$\tan A + 2 \cot 2 A = \sin A \left(1 + \tan A \tan \frac{A}{2}\right).$$

3. Given the direction and velocity of projection, find the direction and velocity of a projectile at the end of t'', and its height above the horizontal plane passing through the point of projection.

4. A ray of light proceeding from a moveable point is reflected by a fixed plane mirror to a given point; find the locus of the first point when the path of the ray is of given length.

5. Two equal cylinders balance at the extremities of equal arms of a straight lever, when immersed in fluids, whose densities vary as the depth. The surface of one coincides with that of the fluid, and the depth of the upper surface of the other is equal to n times its altitude. Compare the densities of the fluids at equal depths.

6. Given the precession in right ascension of a star, find the corresponding change in the angle of position.

7. If two given spheres touch each other internally, and the interior be taken away, find a point within the remainder such that a particle being placed there shall remain at rest. $\left(\text{Attraction of each particle} \propto \frac{1}{\text{dist}^2}\right)$.

8. Divide a given paraboloid into two parts in the ratio of $m : n$, by a plane inclined at a given angle to the axis.

9. Integrate $y\,dy\,dx = (x + a)\,dy^2 + a\,dx^2$.

10. Find the sum of the m^{th} powers of the reciprocals of the roots of an equation in terms of the inferior powers.

11. A and B are two material points connected by an inflexible rod without weight; B moves on the horizontal plane CB, A descends along the inclined plane AC, the motion of the rod being in a vertical plane. Compare the velocity which A has at the point C with the velocity which it would have were it to descend freely down AC.

12. The equation of the curve made by the intersection of a plane with a surface of the second degree is a quadratic equation.

MONDAY AFTERNOON.—Mr. Hughes.

Fifth and Sixth Classes.

1. If a be greater than b, $a^n - b^n$ is greater than $n\,b^{n-1}(a - b)$, and less than $n\,a^{n-1}(a - b)$.

2. If two equal parabolas have a common axis, a straight line touching the interior and bounded by the exterior will be bisected in the point of contact.

3. Force varies as $\dfrac{b\,A^m + c\,A^n}{A^3}$, find the angle between the apsides.

4. In a triangle whose sides are a, b, c, and A, B, C the

angles opposite, having given B, a and the area of the triangle, find the remaining sides and angles.

5. A body is projected upwards from the lower extremity of a given vertical line with a given velocity; after what time must another be projected downwards from the upper extremity with the same velocity, so as to meet the former in the middle point of the line?

6. A cylinder placed with its axis vertical in a fluid rests with an m^{th} part immersed; when placed in another fluid it rests with an n^{th} part immersed; to what depth would it sink in a mixture composed of equal quantities of these fluids?

7. If the particles of two spheres attract with forces varying as the distance, the force with which the spheres attract each other is as the distance between their centres.

8. Define the centre of a lens, and prove it to be a fixed point.

9. Trace the curve whose equation is $(y-c)^2 = (x-a)^4 (x-b)$, and determine the position of its tangent at the point where $x = a$.

10. Integrate $\dfrac{dx}{(a^2+x^2)^{\frac{3}{2}}}$. $\dfrac{dx}{a+bx+cx^2}$, $a^x x^3\, dx$.

11. Inscribe a semi-circle in a quadrant.

12. If a pole rests with one end on the ground against a wall, and the other attached to a string fixed in the wall, find the tension of the string.

13. The altitude of the mercury in a barometer placed in a given cylindrical diving bell is observed at the beginning and end of a descent; find the depth descended.

14. A body which is half elastic descends along the arc of an inverted cycloid, and is reflected by the axis which is vertical; find the space described in the time of n oscillations of a pendulum vibrating in the same cycloid.

15. Find the height of a lunar mountain.

16. To make a body oscillate in a given hypocycloid.

MONDAY AFTERNOON.—MR. HIGMAN.

Third and Fourth Classes.

1. If a be the approximate square root of any number n, and $n - a^2 = \pm b$, then will

$$\sqrt{n} = a \pm \frac{2\,a\,b}{4\,a^2 \pm b} \text{ very nearly.}$$

2. Prove the properties of the *complemental* triangle; and from these properties, and the expressions for the cosine of an angle in terms of the sides, and for the cosine of a side in terms of the angles, deduce Napier's rules for the solution of right-angled spherical triangles.

3. A and B are two balls whose elasticity $= e$, and A strikes B at rest; prove that if B be infinitely greater than A, A's momentum before impact : momentum communicated to B :: $1 : 1 + e$.

4. If two lines revolving in the same plane round the points S and H, intersect one another in the point P in such a manner that (1), $SP^2 + HP^2 =$ constant quantity: (2), that SP be to HP in the given ratio of n to 1; prove that in each case the locus of the point P is a circle.

5. In the second case of the last problem, suppose rays diverging from the point S, to be incident on a refracting surface, generated by the revolution of the concave circumference of the circle round its axis; then if sin. incidence : sin. refraction :: n : 1, and n be > 1, all rays diverging from S will after refraction diverge *accurately* from H.

6. Find the time of the Sun's passing the vertical wire of a telescope.

7. The lower end of a barometer is immersed in a bason of mercury, and the upper is suspended from the extremity of the beam of a common balance; what weight suspended at the other extremity of the beam, will keep it in equilibrium?

8. If α, β, γ be the angles, which the resultant of three forces, acting at right angles to one another, makes with each of them respectively, then will

$$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1.$$

9. In Newton's second Lemma, if the ordinate vary as the m^{th} power of the abscissa, find the limit of the sum of the areas of the circumscribing parallelograms.

10. Explain the nature of centrifugal force, and shew that in all curves it $= \dfrac{h^2}{\text{dist.}^3}$ where $h =$ twice the area described in one second of time.

11. Let Q be a point in a semi-circle whose diameter is AB, join AQ and in AQ produced, take AP a mean proportional between AB and AQ; find the equation of the curve which is the locus of the point P; its area; the content of the solid generated by its revolution, and the radius of curvature at its vertex.

12. Prove that the curve in the last problem possesses these properties.

(1) That a particle placed any where in its perimeter as at P, will attract a particle at A, in the direction AB, with a constant force; supposing the force of attraction to vary as $\dfrac{1}{\text{dist.}^2}$.

(2) That if it be made the revolving orbit in the ninth Section of Newton, and $G = \dfrac{F}{2}$, the orbit traced out in fixed space will be the lemniscate of Bernoulli.

MONDAY EVENING.—Mr. Higman.

1. If a right cone whose vertical angle is 90°, be cut by a plane which is parallel to one touching the slant side, prove that the latus rectum of the section is equal to twice its distance from the vertex.

2. Let a be the distance from the centre of force, from which a body must fall externally to acquire the velocity in a circle whose radius $= r$ when the force $\propto \frac{1}{\text{dist.}}$; and let b be the distance to which it must fall internally to acquire the same velocity; then will a, r, b be in geometric progression.

3. Describe the conic section whose equation is

$$y^2 - 2xy + x^2 - 8x + 16 = 0.$$

4. In what latitude is the angle between the 3 and 4 o'clock hour lines, on a horizontal dial, a maximum?

5. If when the mercury in a true barometer stands at an altitude a, the mercury in an imperfect one of given length stand at the altitude b; what will be the height of the mercury in the true barometer, when it stands at an altitude c in the imperfect one?

6. Deduce the equation to the catenary,

$$2y = a\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right),$$

where $a =$ tension at the lowest point; and prove that its radius of curvature is equal to its normal.

7. If at the distance a from the centre of force, a body be projected at an angle of 45° with the distance, with a velocity which is to that in a circle at the same distance as $\sqrt{2}$ to $\sqrt{3}$; and the force $= \frac{2a^2 n}{\text{dist.}^5} + \frac{n}{\text{dist.}^3}$, required the curve described.

8. In an ellipse find the locus of the intersections of the lines SP and CQ, which cut off the true and eccentric anomalies.

9. If the sides of a triangle ABC be bisected in the points D, E, F; then the centre of the circle inscribed in the triangle DEF, is the center of gravity of the perimeter of the triangle ABC.

10. The sum of the squares of the co-efficients of the expansion of $a^t = \frac{1}{\pi} \int e^{2k\cos x}\, dx$ taken between $x = 0$, and $x = \pi$; k being $= \log a$.

11. If the radius vector and perpendicular on the tangent in any curve described by a revolving body, be denoted by r and p; and in the curve of a star's apparent aberration, as seen from this body, by r' and p', then will $r : p :: r' : p'$.

12. If a prism be laid on its base in the open air, and the eye be placed in a proper position; the base will appear to be divided into two parts, the one much brighter than the other, and separated from one another by a coloured bow, concave towards the eye. Give Newton's explanation of the cause of this phenomenon.

13. Let $\frac{A'}{A}, \frac{B'}{B}, \frac{C'}{C} \ldots$ be the 1st, 2d, 3d, ... approximations to the value of a fraction, when the continued fraction terminates; then will the fraction

$$= \frac{A'}{A} + \frac{1}{AB} - \frac{1}{BC} + \frac{1}{CD} - \frac{1}{DE} + \ldots$$

14. Sum the series:

$$\frac{1}{1.4} + \frac{1}{2.5} + \frac{1}{3.6} + \&c. \text{ to } n \text{ terms:}$$

$$\frac{1}{1.2} - \frac{3}{4.5} + \frac{5}{7.8} - \&c. \text{ to infinity;}$$

$$\frac{1}{1^5} - \frac{1}{3^5} + \frac{1}{5^5} - \frac{1}{7^5} + \&c. \text{ to infinity.}$$

15. Find the integrals of

$$\frac{(x^3 + x^2 + 2)\,d\,x}{x^5 - 2\,x^3 + x}\,;\ a^x \sin^3 x\,d\,x\,;$$

and the relation between x and y in the equation

$$\frac{d^4 y}{d\,x^4} - 2\frac{d^3 y}{d\,x^3} + 2\frac{d^2 y}{d\,x^2} - 2\frac{d\,y}{d\,x} + y = 0.$$

16. Prove that the area of a regular polygon *inscribed* in a circle, is a *geometric* mean between the areas of an inscribed and of a circumscribed regular polygon of half the number of sides; and that the area of a regular polygon *circumscribed* about a circle, is an *harmonic* mean between the areas of an inscribed regular one of the same number of sides, and of a circumscribed regular one of half that number.

17. In what latitude will a ring surrounding the earth, and parallel to the equator, attract a particle placed in the earth's centre with the greatest force possible?

18. A body acted on by gravity moves on the surface of a cone, whose axis is vertical; find the law of force tending to the vertex, by which the projection of its path on a horizontal plane passing through the vertex may be described; and hence deduce the angle between the apsides when the orbit is nearly circular.

19. If the radius of the anterior surface of a concave glass speculum of inconsiderable thickness $(c) = a$; then if the radius of the second surface $= a + \frac{13\,c}{9}$, the image of a distant object formed by reflection at the first surface will coincide with the image formed by reflection at the second surface, and by refraction at the first.

20. A rod of given length and weight, and of uniform density rests with one end in water, and the other on the edge of the vessel which contains it, find the magnitude of the part immersed, and the pressure on the side of the vessel.

21. If $\alpha, \beta, \gamma, \ldots\ldots$ be the roots of the equation $X = 0$, and $A, B, C \ldots$ the results when these roots are substituted for x in the limiting one; then will

$$y = \frac{X}{A\,(x-\alpha)}\,a + \frac{X}{B\,(x-\beta)}\,b + \frac{X}{C\,(x-\gamma)}\,c + \ldots$$

be the equation of the parabolic curve which passes through the points of which $\alpha, \beta, \gamma \ldots$ are the abscissas, and $a, b, c \ldots$ the corresponding ordinates.

22. If a circle whose diameter is equal to the whole tide in any given latitude be placed vertically, and so as to have the lower extremity of its diameter coincident with the level of low water, prove that the tide will rise or fall over equal arcs in equal times.

23. From a bag containing two balls, a white ball is drawn twice following, the ball having been replaced in the bag after the first drawing; required the probability that both balls are white, and that the ball being a second time replaced, a white ball will be drawn at the third trial.

24. If a body, whose elasticity $= m$, descend from rest through an altitude h, by the uniform force of gravity, in a medium whose resistance to a velocity $v = gkv^2$, and impinging on a horizontal plane, rise and fall alternately; prove that the whole space described by the body

$$= \frac{1}{g\,k^2} \log \left\{ \frac{m^2\,e^{-2gk^2h} - 1}{m^2 - 1} \right\}.$$

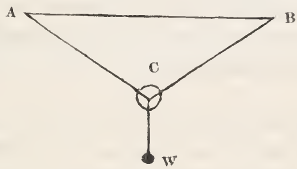
TUESDAY MORNING.—Mr. Higman.

First and Second Classes.

1. Prove that $\log x = n\,(x^{\frac{1}{n}} - 1)$ nearly, when n is very great.

2. If two lines SP, HP revolve about the points S, H, so that $SP \times HP = CS^2$, (C being the middle point of SH) then the locus of the point P is the Lemniscate of Bernoulli.

3. A and B are two given points in a horizontal line, to which are fastened two strings of given lengths: the string BC



passes through a ring at C, and is fastened to a given weight W; find the position in which the weight will rest.

4. A wine glass in the form of a paraboloid, is partly filled with water, and then inverted on a table; given the weight of the glass, required the greatest quantity of water that can be contained without running out.

5. If δ = distance of a plane from the origin of the three rectangular co-ordinates AX, AY, AZ; and if α, β, γ be the angles which δ makes with these three respectively, then will the equation to the plane be

$$x \cos \alpha + y \cos \beta + z \cos \gamma = \delta.$$

6. Supposing a Comet of the same magnitude and density as the Moon, on its nearest approach to the Earth, to be distant thirty radii from the Earth's centre; required the magnitude of the tide raised by the Comet.

7. Describe the experiment by which Newton shewed that the more refrangible rays are the more reflexible.

8. If a body describe the spiral of Archimedes, the force being in the pole, and its motion beginning from that point; then will the times of the successive revolutions be as the differences of the cubes of the natural numbers; and the excess of the time

of the $\overline{n+1}^{\text{th}}$ revolution above that of the $n^{\text{th}} = n \times$ excess of the 2^{nd} above the 1^{st}.

9. If S be the Sun, and A, B two planets that appear stationary to one another; then tan SBA : tan SAB :: periodic time of A : periodic time of B.

10. If $N =$ n^{th} term of the expansion of a^x, determine n when the series reckoned from that term, begins to converge; and shew that the sum of all the terms which follow N is less than $\dfrac{Nn}{1 - x \log a}$.

11. If a be an approximate value of x in any equation, and b, c be the results, when a is substituted for x in the original and in the limiting equation; then will $x = a - \dfrac{b}{c}$ nearly.

12. If any number of forces $p, q, r \ldots$ in different planes, acting on a point make the angles $\alpha, \beta, \gamma \ldots$ with the resultant, then will

$$p \cos \alpha + q \cos \beta + r \cos \gamma + \ldots \text{ be a maximum.}$$

TUESDAY AFTERNOON.—Mr. Higman.

Fifth and Sixth Classes.

1. Prove geometrically, that the side of a square is incommensurable with its diagonal.

2. Draw a line perpendicular to two straight lines not in the same plane.

3. If c be the hypothenuse of a right-angled spherical triangle, prove that

$$\sin^2 \frac{c}{2} = \sin^2 \frac{a}{2} \cos^2 \frac{b}{2} + \cos^2 \frac{a}{2} \sin^2 \frac{b}{2}.$$

4. In a parabola find the area included between the curve, its evolute, and its radius of curvature.

5. A given weight P draws another given weight W up an inclined plane of given height and length, by means of a string parallel to the plane; when and where must P cease to act that W may just reach the top?

6. Prove that the chord of curvature $= \dfrac{PQ^2}{QR}$.

7. In a given latitude, on a given day of the year, determine the time during which a primary, and secondary rainbow may be seen.

8. Investigate the expression for the area of a plane triangle in terms of the sides; and apply the result to the case where the sides are 24, 30, 18.

9. Find x so that $\dfrac{\tan^3 x}{\tan 3x}$ may be a maximum.

10. Find the integrals of $\dfrac{x^6\,dx}{1+x^2}$; $\dfrac{x^{\frac{1}{2}}\,dx}{\sqrt{(2a-x)}}$; $\dfrac{dx}{\sin x}$.

11. If the periodic times of a body revolving in a circle round the centres of force S and R be the same, compare the force tending to S with that tending to R.

12. Find the centre of oscillation of an isosceles triangle, vibrating about an axis which is perpendicular to its plane, and passes through the angle contained by the equal sides of the triangle.

13. A circle whose plane is vertical, is just immersed in a fluid; divide it by a horizontal line into two such parts, that the pressures on them may be equal.

14. On a given day, in a given latitude, the Sun being on the meridian, determine geometrically the angle at which a rod of given length must be inclined to the horizon, that its shadow may be the greatest possible.

15. Transform 8978 from a local value 11, and 3256 from a local value 7, to a system in which the local value is 12; and multiply the numbers together in that system.

16. The image of a circular arc concentric with the reflector, subtends the same angle as the object both from the surface and the center.

17. If PSp be any line drawn through the focus S of a conic section, meeting the curve in the points P and p, and SL be the semi-latus rectum, then will SP, SL, Sp be in harmonic progression.

18. If P' be a point so taken in the radius vector SP of a parabola, that $SP' = SY$ the perpendicular on the tangent, then will the locus of the point P' be the elliptic spiral; prove this, and compare the times of two bodies describing AP and AP', the absolute forces being the same in both cases.

TUESDAY AFTERNOON.—Mr. Hughes.

Third and Fourth Classes.

1. D is a point within a triangle ABC; having given AB, AC and the angles ABD, ACD, BDC, find BC.

2. Describe a circle about a given segment of a parabola made by an ordinate perpendicular to the axis.

3. Compare the pressures on the upper and lower halves of a hemispherical vessel filled with fluid.

4. If (i) be the angle of incidence of a ray passing through a prism in a plane perpendicular to its axis, (e) the angle of emergence, (a) the vertical angle of the prism, and $1 : n :: \sin I : \sin R$ out of the ambient medium into the prism, then

$$\sin e = \frac{1}{n} \sin a \sqrt{(1 - n^2 \sin^2 i)} - \cos a \sin i.$$

5. A hemispherical vessel rests with its base on a horizontal plane; having given the weight and inner radius of the basin, find the specific gravity of a fluid which when just filling it shall begin to run out at the bottom.

6. Find the velocity acquired by an inelastic body descending through a system of three planes, the first being vertical, the second inclined at 45° and the third at 15° to the horizon, respectively.

7. Find the isosceles triangle of a given area, which vibrating about an axis passing through its vertex perpendicular to its plane shall oscillate in the least time possible.

8. Integrate

$$x^2\,dx\,(\log x)^2,\ \frac{x^3\,dx}{\sqrt{(x^2+1)}}\ \text{and}\ dy + \frac{n\,y\,dx}{\sqrt{1+x^2}} = a\,dx.$$

9. Prove that $(1+x)^m$ may in all cases be expressed by a series of the form

$$1 + ax + bx^2 + \&c.$$

10. The altitudes of two stars as they cross the prime vertical are observed, and the difference of their right ascensions is known; find the latitude of the place.

11. If a body describe an oval round a centre of force, the distance at which the angular velocity is equal to the mean angular velocity is $\sqrt{\frac{A}{\pi}}$, where A is the area of the figure, and $\pi = 3.14159$.

12. Having given the centre of gyration of a circle revolving about a diameter, find the centre of gyration of an ellipse revolving about either axis.

TUESDAY EVENING.—Mr. Hughes.

1. Having given the area, the base and the sum of the angles at the base, find the difference of the angles at the base.

2. A vertical cylindrical tube is connected by a horizontal branch with a cubical vessel of water, and the water is made to ascend in the tube by a condensing syringe applied to the top

of the vessel. Having given the dimensions of the vessel, tube and syringe, and the elevation of the water in the tube and vessel; find the number of descents of the piston.

3. A body half elastic moving along a horizontal plane is reflected by a hard plane inclined at an obtuse angle to its course; prove that the time of flight on the inclined plane : time of acquiring the velocity before impact by a body descending vertically :: tangent of plane's inclination : radius.

4. If u be a function of x, and in the equation $\frac{du}{dx} = 0$, there be m roots equal to a and n roots equal to b, then there will be one minimum value of u for each of the roots a and b if m and n be odd, and neither maxima nor minima values when they are even.

5. If an object be placed between two plane reflectors inclined at any angle; find the locus of an eye such that the length of the ray by which any given image is seen may be equal to a given line.

6. If $\frac{A}{B}$ be a fraction in its lowest terms, B greater than A, and of the form $B' \cdot 2^m \cdot 5^n$; the quotient will be a mixed circulating decimal, and the higher of the indices m, n will be the number of figures in the part which does not recur.

7. ACB is a quadrant of a circle whose centre is C, CA, CB its radii, AD, BE equal arcs, DE the chord of the arc DE; shew that the solid generated by the revolution of the circular segment DE about either radius is equal to twice the sphere whose diameter is $\sqrt{2} \sin \frac{1}{2} DE$.

8. An imperfectly elastic body revolving in an ellipse whose eccentricity is $\frac{1}{2}$ is reflected at the mean distance by a plane coincident with the distance, so as to move after impact in the direction of the axis minor; find the degree of elasticity, and compare the periodic times in the two ellipses $\left(F \propto \frac{1}{D^2}\right)$.

9. A cylindrical tube is filled with fluid and closed at both ends; compare the pressures on its sides at the Earth's surface and at a given altitude above it, supposing the bulk of the fluid from change of temperature to be diminished an n^{th} part, and the axis to be vertical in both positions.

10. State how the Sun, Planets and fixed Stars are affected by aberration; and shew that the part of the aberration arising from the motion of the planet varies as $\frac{\cos SPT}{\sqrt{SP}}$, S being the Sun, T the Earth and P the Planet.

11. If particles of a spherical shell attract with forces varying as $\frac{1}{D^2}$, and a cylindrical rod of uniform density whose length equals n times the radius of the sphere pass through the shell; find the pressure on the shell when the rod is at rest, the part of it within the shell being equal to the radius of the sphere.

12. From the vertex of a parabola a straight line is drawn inclined at 45° to the tangent at any point; find the equation to the curve which is the locus of their intersections.

13. If $x^n + A\,x^{n-1} \ldots - P\,x^p \ldots - S\,x^s \ldots + T\,x + V = 0$, where P is the greatest and S the last negative co-efficient, then $\dfrac{V^{\frac{1}{s}}}{V^{\frac{1}{s}} + P^{\frac{1}{s}}}$ is an inferior limit of the positive roots.

14. Integrate

$$\frac{dx}{(1-x)(1-2x)^{\frac{1}{2}}},\quad \frac{dx}{(x-a)\,x^{\frac{2n+1}{2}}}\quad \text{and } x^2\,d^2y = x\,dx\,dy + n\,y\,dx^2.$$

15. A body acted on by gravity ascends along the concave part of a vertical semi-circle from the extremity of the horizon-

tal radius; find its initial velocity so that after quitting the circumference it may pass through the centre.

16. Find the centre of gyration of a right cone revolving about an axis passing through its centre of gravity parallel to one of its sides.

17. Find the horary motion of the Moon's nodes in a circular orbit.

18. Find the correction due to the length of a pendulum for the thickness of its axis.

19. If the inscribed sphere be taken away from the Earth, find the time in which a particle situated in the plane of the Earth's equator within the space occupied by the inscribed sphere will reach the inner surface of the remaining meniscus; the Earth being supposed an oblate spheroid of small ellipticity.

20. If I be the obliquity of the ecliptic, L the Sun's longitude, A his right ascension, then

$$A = L - \left\{ \tan^2 \frac{I}{2} \cdot \frac{\sin 2L}{\sin 1''} - \tan^4 \frac{I}{2} \cdot \frac{\sin 4L}{\sin 2''} + \tan^6 \frac{I}{2} \cdot \frac{\sin 6L}{\sin 3''} - \&c. \right\}.$$

21. Q is the focus of incidence of a pencil of rays which passes nearly perpendicularly through the sides of a prism whose vertical angle A is small, q the focus of emergent rays, and QC, qc perpendicular to the first and second surfaces respectively. Having given the ratio of $\sin I : \sin R$ out of the ambient medium into the prism, and also QC, CA and the angle A, find qc and Ac.

22. P and Q are two material points connected by an inflexible line PQ; P moves along a groove and PQ on a smooth horizontal plane. Having given the initial position of the rod and the quantity and direction of the motion communicated to P; find the angular velocity of the rod when it coincides with the groove.

23. Resistance varies partly as the velocity and partly as the velocity squared; construct for the time when a body is projected downwards with a velocity greater than the greatest it can acquire from rest.

24. Every surface of the second degree may be generated by a circle of variable radius moving parallel to itself, the centre moving along a diameter of the surface. Prove it in the case of the ellipsoid.

1825.

MONDAY MORNING.—Mr. Chevallier.

First and Second Classes.

1. Represent a million according to the duodenary scale of notation.

2. Divide 29^2 into two other square integer numbers.

3. Required the sum of

$$\frac{1^2}{2.3.4.6} + \frac{2^2}{3.4.5.7} + \text{\&c. to } n \text{ terms.}$$

4. If A_1 is the area of a given logarithmic spiral from a distance r to the centre, A_2 the corresponding area of the curve traced out by the perpendicular upon its tangent, A_3 that of the curve traced out by the perpendicular on *its* tangent, and so on continually; find the value of

$$A_1 + A_2 + A_3 + \text{\&c.}$$

5. A seconds' pendulum is carried to the height of one radius above the earth, and another is sunk to the depth of half a radius. Compare the times of their oscillations.

6. If the latitude of a place be determined by observing the altitude of the Sun at 6 o'clock, and the tabulated declination be affected by a small error, find the corresponding error in latitude.

7. A weight P is *supported* by a string, which passes over a fixed pulley, is wound several times round a cylinder Q and

attached to a pin on the other side, the strings being all parallel. If the string be now cut between the cylinder and this point of support, required the motions of P and Q.

8. Given the radius of the Moon and her mass compared with that of the Earth, to find the density of the atmosphere at the Moon's surface, supposing it to be similar to our own.

9. Shew that a sphere is emptied through a small orifice at the lowest point in less time than any other spherical segment of the same capacity.

10. A flood-gate moves upon a vertical axis, the area on one side of the axis being the quadrant of a circle, and on the other side a parallelogram of the same altitude. Required the width of the parallelogram so that the gate may *just* open by the pressure of the water when it has risen to the top.

11. Given the distance of Jupiter from the Sun, his radius and the times of his diurnal and annual revolutions, to compare the aberration of a given Star when it passes the meridian of an observer in his equator at mid-day and at mid-night.

12. Prove that, in the direct impact of perfectly hard bodies, the difference of the *vires vivæ* before and after impact is equal to the sum of the *vires vivæ* of the bodies moving with the velocities lost and gained respectively.

MONDAY AFTERNOON.—MR. CHEVALLIER.

Fifth and Sixth Classes.

1. Required the present worth of £75. due 15 months hence at 5 per cent. per annum.

2. Shew that, if the sum of the digits in the odd places be subtracted from any number expressed in decimal notation, and the sum of the digits in the even places be added to the same number, the result is divisible by 11.

3. The difference of the means of four numbers in geometrical progression is 2 and the difference of the extremes is 7. Required the numbers.

4. Prove that $\sin 75^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$ to rad. 1.

5. Find $\tan 5A$ in terms of $\tan A$.

6. Shew that if two circles cut each other, and from any point in the straight line produced which joins their intersections, two tangents be drawn, one to each circle, they shall be equal to one another.

7. Solve the equation $x^3 - 6x^2 + 11x - 6 = 0$, the roots being in arithmetical progression.

8. Integrate

$$\frac{dx}{x^3\sqrt{(1-x^2)}}; \quad \frac{dx}{x^2 - 5x + 6}.$$

9. Trace the spiral in which $r = a\cos\theta$, and find its area between the values $\theta = 0$ and $\theta = \dfrac{\pi}{2}$.

10. If P and Q are two weights connected by a string passing over a fixed pulley, and acting in directions parallel to two given inclined planes having a common altitude, and P descends, find the space described in t'' and the velocity acquired.

11. Explain the construction of a Fire-engine.

12. Compare the resistance upon the surface of a cone moving in a fluid with a given velocity in the direction of its axis, with the resistance upon its base.

13. In a given latitude, find the altitude of the Sun on the day of the equinox at 9 in the morning.

14. Trace the image of an indefinite straight line *in contact* with a spherical reflector.

15. Find the space through which a body must fall externally that it may acquire the velocity with which it moves in an ellipse about the *centre*.

16. If the mass of a planet is four times that of the Earth, and the distance of its satellite 16 times that of the Moon from the Earth, in how many months will the satellite revolve?

17. Find the position of the centre of gravity of the area of a semi-parabola.

18. Find the time in which a straight line of given length will oscillate when suspended at its extremity.

MONDAY AFTERNOON.—MR. WARREN.

Third and Fourth Classes.

1. Find one of the roots of the equation $3x^3 - 26x^2 + 34x - 12 = 0$ by the method of divisors.

2. In a parabola, in the focal distance SP, Sp is taken equal to the ordinate PN. Find the equation to the curve traced out by the point p.

3. If a body oscillate in a cycloid beginning at the highest point, the tension of the string at any point arising from the centrifugal force equals the tension arising from gravity.

4. Find the time in which a vessel formed by the revolution of a cycloid about its axis, placed with its axis vertical and its vertex downwards, will empty itself through a small orifice at the vertex.

5. Find the point of contrary flexure of a spiral, where the angle varies inversely as the n^{th} power of the radius vector.

6. Express a circular arc in a series in terms of its tangent.

7. When the centripetal force varies inversely as the n^{th} power of the distance, n being greater than 3; find the equation to the spiral, which a body, projected with a velocity equal to the velocity acquired by falling from an infinite distance, describes; and determine the number of revolutions which it makes, before it falls into the centre.

8. If v represent the true anomaly of a planet, reckoning from the perihelion, u the eccentric anomaly and e the eccentricity; when e is small, $v - u = e \sin u$, nearly. Required proof.

9. Compare the mean addititious force with the force by which P is retained in its orbit round T. (*Newton*, Prop. 66. Cor. 17.)

10. State the phænomena, from which it appears that the force, by which the Moon is retained in its orbit, tends to the Earth, and that this force varies inversely as the square of the distance.

11. Find the centre of gyration of an hyperboloid revolving about its axis.

MONDAY EVENING.—Mr. Warren.

1. The present value of an annuity of 1£. to continue x years is 10£. and the present value of an annuity of 1£. to continue $2x$ years is 16£. What is the rate of interest?

2. The roots of the equation $x^n - p\,x^{n-1} + q\,x^{n-2} - r\,x^{n-3} + \&c. = 0$ are in geometrical progression beginning from unity; given $p = 15$, $q = 70$. Required n, r, &c.

3. If a, b, c, d be the sides of a quadrilateral inscribed in a circle, and $S = \dfrac{a+b+c+d}{2}$; prove that the area of the quadrilateral

$$= \sqrt{(S-a)\,(S-b)\,(S-c)\,(S-d)}.$$

4. Express the chord of 36° to radius unity in a continued fraction.

5. A body whose elasticity : perfect elasticity :: e : 1, projected from a horizontal plane, at a given elevation, with a given velocity, impinges against a perfectly hard vertical plane, whose distance from the point of projection is given. Required the

horizontal range of the body; the vertical plane being supposed perpendicular to the plane of the body's motion.

6. Explain the nature of the stereographic projection of the sphere, and shew that the projections of all circles, the planes of which do not pass through the eye, are circles.

7. A body describing a parabola, by a uniform force acting in parallel lines, receives an impulse. Given the line which the body would describe by the impulse alone in a given time T; and the chord of the parabolic arc, which it would have described in the same time T if no impulse had been communicated to it: to find the chord of the arc which it actually does describe.

8. Find the area of the least parabola which can circumscribe a given circle.

9. A solid of revolution, whose axis is perpendicular to the horizon, empties itself through a small given orifice. Required its nature when the velocity of the descending surface is uniform.

10. A paraboloid, generated by the revolution of a parabola, whose equation is $y^n = a^{n-1} x$, placed with its vertex downwards and its axis vertical, empties itself through a small orifice at the vertex; and the value of n is such, that if a sphere empty itself in the same time that the paraboloid does, half the sphere will empty itself in the same time that half the paraboloid does: compare the distance of the descending surface from the vertex when half the paraboloid is emptied with the distance at first.

11. Sum the series

$$\sin a + \sin (a + b) + \sin (a + 2b) + \text{\&c. to } n \text{ terms.}$$

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \text{\&c. to infinity.}$$

12. Determine the position of equilibrium of a uniform rod, one end of which rests against a plane perpendicular to the horizon, and the other on the interior surface of a given hemisphere.

13. Supposing the force of gravity to vary inversely as the n^{th} power of the distance; by what law does the density of the atmosphere vary?

14. Shew that the length of a parabola, whose equation is $a y^{2n} = x^{2n+1}$, where n is any whole positive number, may be found in terms of the abscissa in a finite algebraical form.

15. Shew how the polar equation to caustics formed by reflection may be found generally; and apply the method when the reflecting curve is a circular arc, and the radiating point in the circumference of the circle.

16. Find the equation to the curve surface in which the tangent plane at any point intersects the axis of z at a distance from the origin equal to m times the corresponding value of z.

17. A corpuscle placed within a circle is attracted to every particle in the circumference with a force that varies inversely as the square of the distance: prove, when the distance of the corpuscle from the centre is small, that the attraction on the corpuscle varies nearly as its distance form the centre, and draws it from the centre.

18. In the 10th Lemma of the 1st Section of Newton, where the abscissa AD represents the time, the ordinate DB the velocity, and the area ABD the space described; if a straight line be drawn touching the curve AB in B the extremity of the ordinate, the tangent of the angle which this line makes with the axis will represent the force Required proof.

19. An arch, where the equilibrium is preserved by the weights of the voussoirs, is so constructed, that the centres of gravity of the voussoirs are in a catenary curve and the joints perpendicular to the curve. Required the equation from which the length of the voussoirs may be obtained.

20. Investigate the formula of lunar nutation in right ascension; find the longitude of the Moon's node when it is a maximum, and thence its maximum value; and by means of these values express the nutation in right ascension in a more simple formula.

21. Given the ratio of the periodic time of the Moon to the time of the Earth's revolution about its axis, and the ratio of the mean distance of the Moon to the mean semi-diameter of the Earth, to find the ratio of the polar and equatoreal diameters of the Earth nearly.

22. A weight P raises up another weight Q by means of a string passing over a fixed cylindrical pulley; given P, Q and the weight of the pulley, to compare the tensions of the two parts of the string: the weight of the string and the friction at the axis of the pulley being neglected.

23. A body oscillates in a cycloid in a resisting medium, where the resistance is as the square of the velocity; find the resistance at any point of the body's motion, (Newton, Prop. 29, Book II.)

24. Prove from the preceding proposition; if a space S be taken so as to be represented by the rectangle of the hyperbola in Newton's figure on the same scale that the cycloidal arc is represented by the hyperbolic area, and V be the velocity acquired in falling through S by the force of gravity *in medio non resistente;* that V is the limit of the velocity which can be acquired in the resisting medium by the force of gravity.

TUESDAY MORNING.—Mr. Warren.

First and Second Classes.

1. Draw a perpendicular to two given straight lines not in the same plane.

2. Sum the series

$$1 + 2^2 x + 3^2 x^2 + 4^2 x^3 + \text{\&c. to } n \text{ terms.}$$

$$\frac{x}{1 \cdot 2} + \frac{x^2}{2 \cdot 3} + \frac{x^3}{3 \cdot 4} + \text{\&c. to infinity.}$$

3. Express $\cos nA$ in terms of $\cos A$, n being a whole number.

4. When a chain fixed at two points is acted upon by a central attractive or repulsive force, the tension at any point is inversely as the perpendicular let fall from the centre of force on the tangent at that point. Required proof.

5. Determine the point in the curve surface on which a semi-paraboloid will rest on an horizontal plane.

6. Having given the refracting powers of two mediums; find the ratio of the focal lengths of a convex and concave lens formed of these substances, which when united produce images nearly free from colour.

7. Compare the time in which a sphere slides down an inclined plane with the time in which it rolls down the same plane.

8. A given uniform rod moves in the same plane in a hemisphere. Determine its motion.

9. Shew how the complete integral of the differential equation

$$A y + B \frac{dy}{dx} + C \frac{d^2 y}{dx^2} + D \frac{d^3 y}{dx^3} + \&c. = X,$$

is to be obtained, when A, B, C, D, &c. are constant quantities, and X a function of x.

10. Given the heights of the spring and neap tides, to compare the densities of the Sun and Moon; their apparent diameters being considered as equal.

11. Explain the cause of aberration of light; shew how it is to be measured, and distinguish accurately between the aberration of the fixed stars and the aberration of the planets.

12. Determine generally the resistance of a medium, so that a body acted upon by a centripetal force, whose law is known, may move in a given curve; and thence find the resistance when the force is uniform and acts in parallel lines, so that the curve may be a circular arc.

TUESDAY AFTERNOON.—MR. WARREN.

Fifth and Sixth Classes.

1. If the terms of an equation, all whose roots are possible, be multiplied by the terms of the arithmetical progression 0, 1, 2, 3, &c. the resulting equation will be a limiting equation to the former, with this exception, that no root of the limit will lie between the positive and negative roots of the proposed equation.

2. The centre of an ellipse coincides with the vertex of a common parabola, and the axis major of the ellipse is perpendicular to the axis of the parabola. Required the proportion of the axes of the ellipse, so that it may cut the parabola at right angles.

3. Several bodies are projected from a given point, with the same velocity, in different directions, being acted upon by the force of gravity. Find the locus of them all at the end of a given time.

4. Explain the method of drawing asymptotes to spirals, and apply it to the hyperbola considered as a spiral having the pole in the focus.

5. Find the elasticity of two bodies A and B and their proportion to each other, so that, when A impinges upon B at rest, A may remain at rest after impact and B move on with an n^{th} part of A's velocity.

6. Given two sides of a triangle and the difference of the angles opposite to them. Required the other angle.

7. Find the geometrical focus of a small pencil of rays diverging from a given point in the axis and incident nearly perpendicularly on an elliptic reflector.

8. Determine at what angle the wind must strike against the sails of a mill, so that the effect to put them in motion may be the greatest possible.

9. Given the latitude of the place and the declination of the Sun. Find the time that the Sun is above the horizon.

10. Explain the principal advantages of Briggs' system of logarithms.

11. The equation $x^3 - 7x^2 + 16x - 12 = 0$ has two equal roots. Find all the roots.

12. Find the law of the force tending to the pole of the logarithmic spiral.

13. The sum of the three angles of a spherical triangle is greater than two right angles and less than six right angles. Required proof.

14. The whole surface of a cone is three times the area of the base. Find its vertical angle.

15. When the force varies inversely as the n^{th} power of the distance, compare the velocity acquired by falling from an infinite distance with the velocity of a body revolving in a circle.

16. Given the hypothenuse of a right-angled triangle and the side of an inscribed square. Required the two sides of the triangle.

17. Investigate the differential expression for the radius of curvature, and apply it to find the radius of curvature of the logarithmic curve.

TUESDAY AFTERNOON.—Mr. Chevallier.

Third and Fourth Classes.

1. What is the present value of a freehold estate of 150£. a year, allowing the purchaser 6 per cent. compound interest?

2. Shew that the greater two consecutive numbers are, the less is the difference between their logarithms.

3. Compare the curvatures of an ellipse at the extremities of the major and minor axes.

4. A given weight is to be supported at a given point upon a straight lever of uniform density by a power acting at its extremity, on the same side of the fulcrum. Required the least power which will support the system, and the corresponding length of the lever.

5. Find the point in a given hyperbola where the velocity of a body acted on by a force tending to its focus is twice as great as the velocity in a parabola at the same distance.

6. If a solid cylinder and a thin hollow cylinder of the same *weight* and radius roll together from rest down a given inclined plane, how far will they be separated after a given time?

7. If the Earth's motion about its axis were to cease, how much would a clock keeping true time in a given latitude gain in 24 hours?

8. Given the time of Sun-rise and the altitude of the Sun when due east on the same day, to find the latitude of the place and the declination of the Sun.

9. State what advantage is obtained by substituting a glass rectangular prism for a plane reflector, in the construction of Newton's telescope: and trace the course of a pencil of rays on that supposition.

10. A spherical bubble composed of matter the specific gravity of which is S, and filled with gas of the specific gravity s, just floats in air, specific gravity σ. Required the thickness of the bubble.

11. Solve the equation $x^6 + 1 = 0$.

12. Given the declination of the Moon, to find the duration of the superior or inferior tide occasioned by her action alone.

TUESDAY EVENING.—Mr. Chevallier.

1. The sum of two numbers is 6 and the sum of their cubes 72. Required the numbers.

2. Upon a given straight line as an hypothenuse, describe a right-angled triangle which shall have its three sides in continued proportion.

3. Find a series of fractions converging to $\frac{41}{72}$.

4. Sum the following series:

$$\frac{1}{2} + \frac{3}{4} + 1 + \&c. \text{ to } 8 \text{ terms.}$$

$$\frac{1}{1\,.\,3} + \frac{1}{2\,.\,4} + \frac{1}{3\,.\,5} + \&c. \text{ to } n \text{ terms.}$$

$$\frac{5}{1\,.\,2\,.\,3\,.\,4} + \frac{7}{2\,.\,3\,.\,4\,.\,5} + \frac{9}{3\,.\,4\,.\,5\,.\,6} + \&c. \text{ to } n \text{ terms.}$$

$$1^3 + 2^3 + 3^3 + \&c. + x^3.$$

5. A circular hoop is supported in a horizontal position, and three weights of 4, 5 and 6 pounds respectively are suspended over its circumference by three strings meeting in the centre. What must be their positions so that they may sustain one another?

6. A cylinder of given altitude has its lower half filled with mercury and the rest with water. Find the time in which it will empty itself through a small orifice in the base.

7. The N. P. D. of α Aquilæ being $81°.\ 38'.\ 25''$ and its observed zenith distance when on the meridian $43°.\ 50'.\ 45''$, find the latitude of the place: and state the several corrections which must be applied to obtain an accurate result.

8. Find the focal length of a double convex glass lens of inconsiderable thickness when the radii of the surfaces are equal, and $\sin I : \sin R :: 3 : 2$.

9. Shew that a horizontal dial, constructed for North latitude l, will be a vertical meridional dial for South latitude $90 - l$.

10. Two bodies are projected from the same point in a horizontal plane with equal velocities and have the same horizontal range. Required the directions of projection so that the area included between the parabolas described may be the greatest possible.

11. Find the time in which a pendulum would oscillate in a hypocycloid within the earth, the diameter of the wheel being half the earth's radius.

12. P descends drawing Q over a fixed pulley. Find the space described in a given time, the string being conceived to have weight.

13. If an ordinate be drawn to the axis of a cycloid from any point P, cutting the circle described upon the axis in D; and from P and D there be drawn tangents to the cycloid and circle intersecting one another in T. Required the curve which is the locus of T, and the area which is contained between it and the cycloid.

14. A straight bar of given length is made to oscillate in its own plane about an axis situated in a line which bisects it at right angles. Required the point of suspension so that the time of oscillation may be the least possible.

15. Integrate the equations,

$$(x^2 + yx)\,dy = (x - y)\,dx.$$

$$(2y^2x + 3y^3)\,dx + (2x^2y + 9xy^2 + 8y^3)\,dy = 0.$$

$$x^3\,dx^3 + dy^3 = ax\,dx^2\,dy.$$

16. If seven balls be drawn from a bag, in which are four white balls and eight black, what is the probability that three white balls will be taken?

17. Required the nature of the curve which cuts perpendicularly a series of similar concentric ellipses.

18. A weight being suddenly removed from the deck of a vessel, the area of which at the surface of the water is given, she is observed to make a small vertical oscillation in t''. Required the weight of the vessel.

19. There is a fixed centre of force which varies inversely as the square of the distance; and about this as a focus an ellipse is described, the axes of which are to one another in the proportion of $\sqrt{2} : 1$. A perfectly elastic body falls from a distance equal to the axis major in the direction of the radius vector passing through the extremity of the axis minor and impinges on the ellipse. Required the motion after impact.

20. The earth being supposed a sphere revolving about its axis with a given angular velocity, find the curve, in a meridional plane, which is the locus of a body, the centrifugal force of which opposed to gravity is every where equal to the force of gravity acting upon it.

21. State the methods by which the masses of all the planets and of their satellites may be compared with that of the Sun.

22. If a body be revolving in an ellipse about the focus, and the force be suddenly made to vary inversely as the cube of the distance, the *actual* force at the mean distance being unaltered, what will be the curve described?

23. Find the curve described by a body projected in a medium, the resistance of which varies as the velocity, and acted upon by gravity.

24. Prove that an arc of a circle, which does not exceed $60°$, is a curve of quicker descent than any other curve which can be drawn within the same arc: and that the arc of $90°$ is a curve of quicker descent than any other curve which can be drawn without the same arc.

1826.

MONDAY MORNING.—Mr. King.

First and Second Classes.

1. Find the equations to a straight line drawn from a given point perpendicular to a given plane.

2. In the cubic equation $x^3 - q\,x - r = 0$ where $\frac{r^2}{4}$ is less than $\frac{q^3}{27}$, shew that the solution may be effected by means of a table of natural sines; and explain why the accuracy of this method cannot be depended upon in practice.

3. In a right-angled spherical triangle whereof c is the hypothenuse and a and b the sides, prove that

$$\tan\frac{c+a}{2}\tan\frac{c-a}{2} = \left(\tan\frac{b}{2}\right)^2.$$

4. Find the time of a small oscillation of an oblique-angled parallelogram vibrating in its own plane about an axis passing through one of its angular points.

5. If from a point in a horizontal plane, any number of bodies be projected in the same vertical plane with such velocities and in such directions, that the areas of the parabolas described shall be equal to one another; find the curve which shall touch them all.

6. Trace the curve whose equation is $y^2 = \dfrac{x^3 - b\,x^2}{x + c}$, and determine the nature of its singular points.

7. A rod of given length and uniform density, is supported in a fluid, the density of which varies as the depth, by a string attached to it at a given point; find the position of equilibrium, supposing one extremity of the rod coincident with the surface.

8. If P be the pole of the heavens, Z the zenith and S a given star, find when the angle ZSP increases fastest.

9. Integrate the following differentials:

$$\frac{dx}{x^4+1}\,;\ e^x \sin mx\,dx\,;\ dy - (x+y)^n\,dx = a^n\,dx.$$

10. Sum the following series:

$$\frac{\sin\theta}{1}\,\frac{1}{2} - \frac{(\sin\theta)^3}{1.2.3}\,\frac{1}{3} + \frac{(\sin\theta)^5}{1.2.3.4.5}\,\frac{1}{4} - \&\text{c. } ad\ infinitum.$$

$$2 + 5 + 24 + 83 + 334 + \&\text{c. to } n \text{ terms.}$$

11. Find the shortest line which can be drawn touching a given ellipse, and intercepted by the tangents drawn at the extremities of the axes of the ellipse.

12. Find the equation to the curve surface in which the tangent plane at any point cuts off from the axis of z a portion equal to the distance of the point of contact from the origin of the co-ordinates.

MONDAY AFTERNOON.—Mr. King.

Fifth and Sixth Classes.

1. Investigate the rule for finding the least common multiple of any two quantities and apply it to find the least common multiple of 174 and 336.

2. Having given the n^{th} term of an arithmetic series and also the sum of n terms, find the series.

3. Having given the sine of 12°, find the sine of 48°.

4. Two circles are situated in the same vertical plane; determine *analytically* and *geometrically* the straight line of swiftest descent from one to the other; and shew that the two results agree.

5. At what hour on a given night, in a given latitude, will the vertical circle passing through a known star cut the equator in a given angle?

6. Find the value of $\frac{\sin 2x + 2\sin^2 x - 2\sin x}{1 - \cos x}$, when $x = 0$.

7. Two straight rods inclined to each other at a given angle are immersed vertically in a fluid in a given position; find the angle formed by their images.

8. Trace the curve whose equation is $a^2 y = \frac{x^5}{x^2 - b^2}$.

9. The resultant and sum of two forces are given, and also the angle which one of them makes with the resultant; it is required to determine the forces and the angle at which they act.

10. Having given two conjugate diameters of an ellipse and the angle contained between them; find the magnitudes and positions of the axes.

11. Find the resistance to a cycloid moving in a fluid in the direction of its base.

12. Enumerate the arguments by which the diurnal rotation of the Earth round its axis and its annual motion round the Sun, are established.

13. Transform the equation $x^3 - p x^2 + q x - r = 0$, whose roots are α, β, γ, into one whose roots are $\alpha^2 + \beta^2$, $\alpha^2 + \gamma^2$ and $\beta^2 + \gamma^2$.

14. Investigate the equation to the orbit in which the centripetal force is always n times as great as the centrifugal, and find the time of one revolution.

15. Determine that point in an ellipse described round a centre of force situated in the focus, where the linear velocity is n times as great as the paracentric.

16. Find the moment of inertia of a rectangle revolving in its own plane, round an axis passing through its centre of gravity.

17. Sum the following series:

$$1 \cdot 2^2 + 2 \cdot 3^2 + 3 \cdot 4^2 + \text{\&c. to } n \text{ terms.}$$

$$\frac{1}{1 \cdot 2} - \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 6} - \frac{1}{7 \cdot 8} + \text{\&c. to infinity.}$$

18. Integrate the following differentials:

$$\frac{(x+4)\,dx}{x^3 - x^2} : \frac{x^4\,dx}{\sqrt{1-x^2}} : \sqrt{x}\,dx + \sqrt{y}\,dx = \sqrt{y}\,dy.$$

MONDAY AFTERNOON.—Mr. Hind.

Third and Fourth Classes.

1. A sum of money P£. is left among A, B and C in such a manner that at the end of a, b and c years, when they respectively come of age, they are to possess equal sums: required the share of each at compound interest.

2. Prove that 1, 3, 5, 7, &c. is the only arithmetical progression beginning from unity, in which the sum of the first half of any even number of terms has to the sum of the second half the same constant ratio; and find that ratio.

3. Shew that vers $(\pi - A) = 2 \text{ vers } \frac{1}{2}(\pi + A) \text{ vers } \frac{1}{2}(\pi - A)$, when the radius $= 1$.

4. Two equal parabolas have a common axis; prove that the area included between one of them and a straight line touching the other is a constant magnitude.

5. From the top of a tower two bodies are projected with the same given velocity at different given angles of elevation, and they strike the horizon at the same place. Find the height of the tower.

6. Determine the magnitude of a sphere of given specific gravity which will rest just immersed in a fluid whose density varies as its depth.

7. Find the principal focus of a reflector generated by the revolution of a cycloid about its axis, and determine the relation between the distances of the conjugate foci from the vertex.

8. Find all the angles in which the curve whose equation is

$$\left(\frac{x}{y}+\frac{y}{x}\right)^2 = 2\,a^2\left(\frac{1}{y^2}-\frac{1}{x^2}\right)$$

cuts the axis, and determine the value of its greatest ordinate.

9. If a body describe a logarithmic curve by a force acting perpendicularly to its axis; prove that the force at any point varies as the body's distance from the axis, and the velocity as the square root of the chord of curvature parallel to it.

10. Integrate $\dfrac{\tan\,\theta\;d\theta}{1-\tan^2\theta}$.

11. Three given points are taken in the circumference of a given circle; find its vertical position on a horizontal plane, that the sum of their altitudes may be the greatest or least possible.

12. In any latitude find when the time of the rising of the Sun's disk bears the least ratio to the time of its crossing the meridian.

13. If a body describe a circle by a force in the circumference, and at the same time the circle revolve about the centre of force in its own plane with an angular velocity

varying inversely as the square of the body's distance; prove that the path traced out in fixed space is the spiral of *Archimedes*, and find the law of force by which it may be described.

14. Construct and graduate a clepsydra in which the weight of the fluid discharged measures the time.

MONDAY EVENING.—Mr. Hind.

1. Express the secant and co-secant of the sum and difference of two arcs in terms of the secants and co-secants of the simple arcs.

2. Shew that the number of permutations of m things taken n together is equal to $m\ (m-1)$ &c. $(m-r+1)$ times the number of permutations of $m-r$ things taken $n-r$ together.

3. In how many different ways is it possible to pay 10£. in crowns, seven shillings and moidores?

4. Two spheres of given magnitudes and elasticity, not affected by gravity, are projected at the same time from given points with given velocities in opposite directions in the same straight line; find *when* and *where* their impact takes place, and their positions at the end of any assigned time after impact.

5. The vertex of a parabola is A and the axis AN, and in the ordinate NP a point Q is taken always equidistant from A and P; find the equation to the curve which is the locus of Q; trace it, and determine the angles in which it cuts the axis and the arcs of the parabola.

6. If t be the length of a part of the catenary, the weight of which is equal to the tension at a point whose abscissa is x and corresponding arc s: prove that

$$s+x : s-x :: \sqrt{t+s} : \sqrt{t-s}.$$

7. The abscissa and double ordinate of a common parabola are a and b, and the diameters of its circumscribed and inscribed circles D and d; prove that $D + d = a + b$.

8. Find the time of emptying the frustum of a cone, the radii of whose ends are R and r and altitude h, through a small orifice in its less base.

9. If a quadrantal arc be divided into two parts in the ratio of $2n : 1$, of which θ is the less; prove that

$$\sin\theta \sin 3\theta \sin 5\theta \text{ \&c. } \sin(2n+1)\theta = \frac{1}{2^n}.$$

10. Required the law of force when the space due externally to the velocity in a circle : space due internally :: $\sqrt{e} : 1$; e being the base of the hyperbolic system of logarithms.

11. The quantities a, b, c, d, &c. are in arithmetical progression; prove that the terms of any order of the differences of the quantities $\frac{a}{b}$, $\frac{b}{c}$, $\frac{c}{d}$, &c. increase or decrease according as the progression decreases or increases.

12. If any number of hyperbolas have a common centre, and at distances proportional to their major axes double ordinates be drawn; shew that bodies acted upon by the same absolute force situated in the centre will describe any of the arcs thus cut off in equal times.

13. Prove that *Des Cartes'* solution of a biquadratic equation succeeds when all the roots are possible and two of them equal, and apply it to solve the equation

$$x^4 - 6x^3 + 8x^2 + 6x - 9 = 0.$$

14. An ellipse whose semi-axes are a and b and eccentricity e, revolves about its major axis: shew that the surface of the solid thus generated is

$$= 2\pi b \left\{ b + \frac{a}{e} A \right\};$$

A being a circular arc whose sine $= e$ to a radius $= 1$.

15. Three straight lines revolve about three given points not in the same straight line and intersect one another in three points; prove that if the loci of two of these intersections be straight lines, the locus of the third will be a conic section.

16. A pencil of parallel rays incident upon a transparent medium bounded by parallel plane surfaces, is partly reflected at the upper surface, partly refracted by the medium, and afterwards reflected at the lower surface; prove that all the rays of the emergent pencil are parallel, and find the angle of incidence that its breadth may be the greatest possible.

17. If the force be repulsive and vary as the distance from the centre of the globe; prove that the oscillations in an epicycloid are isochronous; and having given the radii of the globe and wheel, find the velocity at any point and the actual time of an oscillation.

18. If $\cos(\lambda + \delta) : \cos(\lambda - \delta) :: 1 : 3$, where λ is the latitude of the place and δ the declination of the Moon; prove that the time of the ebbing or flowing of the superior tide : the time of the ebbing or flowing of the inferior tide :: 2 : 1.

19. If the roots of the equation

$$x^n - p\,x^{n-1} + q\,x^{n-2} - \&c. + Q\,x^2 - P\,x + L = 0,$$

be in harmonical progression, then will the greatest and least be respectively

$$\frac{n\sqrt{n+1}\,L}{\sqrt{n+1}\,P - \sqrt{3(n-1)^2 P^2 - 6n(n-1)QL}},$$

and

$$\frac{n\sqrt{n+1}\,L}{\sqrt{n+1}\,P + \sqrt{3(n-1)^2 P^2 - 6n(n-1)QL}}:$$

required a proof.

20. Integrate

$$\frac{(\sec\theta - \tan\theta)\,d\theta}{\sec\theta + \tan\theta} \quad \text{and} \quad \frac{e^\theta \sin m\theta\, d\theta}{1 - 2e^\theta \cos m\theta + e^{2\theta}};$$

and find the relation of x and y in the equation

$$\frac{d^2 y}{d x^2} - a \frac{d y^2}{d x^2} + b x \frac{d y^3}{d x^3} = 0.$$

21. A globe of given weight and magnitude, after descending by the force of gravity a feet in air, passes into another medium whose density is n times as great; required the relations between the space described, the velocity acquired and the time of motion.

22. Find the motions of two equal balls connected by an inflexible rod without weight, one of them being attached to a given weight by means of a string passing over a fixed pulley, and the other moving on a perfectly smooth horizontal plane.

23. If r be the radius vector of an ellipse from the centre, and θ the angle which it makes with the major axis; it is required to express r in a series of the form $A_0 + A_1 \cos\theta + A_2 \cos 2\theta + \&c.$ and to shew particularly how A_0, A_1 and A_2 may be determined.

24. A straight rod which is always parallel to the horizon descends freely by the force of gravity, and at the same time revolves uniformly about one of its extremities; required the equations to the surface traced out by it, and to the tangent plane at any point.

TUESDAY MORNING.—Mr. Hind.

First and Second Classes.

1. Find the sum of the projections of the three sides of a plane triangle upon three planes at right angles to each other.

2. A body is projected perpendicularly upwards and the time between its leaving a given point and returning to it again is given; find the velocity of projection and the whole time of motion.

3. Given a the area corresponding to any rectangular co-ordinates in the figure of secants; to find the content of the solid generated by its revolution round the axis.

4. Sum the series

$$\sin^2 a + \sin^2 (a + b) + \sin^2 (a + 2b) + \text{\&c. to } n \text{ terms.}$$

5. Through any point in the straight line joining the centre and intersection of the tangents to any two points, of an ellipse, two straight lines are drawn respectively parallel to its diameters passing through the points of contact; prove that the triangles formed by these lines and the tangents are equal.

6. If λ be the true latitude of a place, and θ the latitude on *Mercator's* chart constructed to a radius $= 1$; prove that $2 \tan \lambda = e^{\theta} - e^{-\theta}$.

7. A globe of given weight and radius *rolls* down the surface of a hemispherical bowl from rest; find the velocity acquired at any point of its descent.

8. A chain suspended at its extremities from two tacks in the same horizontal line forms itself into a cycloid; prove that the density at any point $\propto \sec^3 (\frac{1}{2} \theta)$, and the weight of the corresponding arc $\propto \tan (\frac{1}{2} \theta)$, θ being the arc of the generating circle measured from the vertex.

9. A small orifice is made in the side of an upright cylindrical vessel, and the vessel revolves about its axis with a given uniform velocity: find the path traced out by the fluid on the horizontal plane.

10. The bias and velocity of projection of a bowl are such as to cause it to describe a given logarithmic spiral; find the direction of projection, so that after having described a path of given length, it may impel the jack in a given direction.

11. Prove that the force by which a body may describe any of the conic sections, round a centre of force in the vertex, varies

inversely as the square of the distance, and directly as the cube of the secant of the angle which it makes with the axis.

12. Find the interval between the heliacal rising and setting of a given star, to a spectator in a given latitude.

13. The velocities of the different parts of a river vary as their distances from the bank; required the path described by a boat moving in a course inclined to the stream at an angle of 45°: also, the velocity at any place and the time of reaching a point at a given distance from the bank.

TUESDAY AFTERNOON.—Mr. Hind.

Fifth and Sixth Classes.

1. Required the discount of 100£. due three years hence, at 5 per cent. per annum, compound interest.

2. Find the values of x, y and z, which satisfy the equations;

$$xy = a(x+y),$$
$$xz = b(x+z),$$
$$yz = c(y+z).$$

3. If a fraction in its lowest terms be converted into a recurring decimal, prove that the number of figures which recur is always less than the denominator of the fraction.

4. Shew that the parameter belonging to any diameter of a parabola varies inversely as the square of the sine of the angle at which the corresponding ordinates are inclined to it.

5. Divide the angle A into two parts, so that their versed sines may be in the given ratio of $m : n$.

6. A body projected in the direction of the action of a constant force, describes P and Q feet in the p^{th} and q^{th} seconds; find the magnitude of the force and the velocity of projection.

7. Trace the curve whose equation is $\sqrt{y} = \dfrac{a}{\sqrt{x}} + \sqrt{x}$, and determine the positions of its asymptotes.

8. A parabola with its axis vertical, has its vertex coincident with the surface of a fluid in which it is immersed; divide it by lines parallel to the surface into four parts, so that the pressures upon them may be equal.

9. Differentiate

$$\frac{a}{a+x} + \frac{b}{b+x} - \frac{a+b}{a-b} \log\left(\frac{a+x}{b+x}\right);$$

and find the value of

$$\frac{a^x \sin a\,x - b^x \sin b\,x}{c^x \text{ vers } c\,x - e^x \tan e\,x}, \text{ when } x = 0.$$

10. Given one of the angles and the perimeter of a plane triangle, to find the sides, when the area is the greatest possible.

11. Prove that the semi-axes major and minor and the semi-latus rectum of an elliptic orbit are respectively an arithmetic, geometric and harmonic mean, between the aphelion and perihelion distances.

12. Integrate $\sin\theta\cos\theta \text{ vers }\theta\, d\theta$.

13. Two bodies begin to descend at the same time down two given inclined planes from given points in the same vertical line; find their distance from each other at the end of any assigned time.

14. In any recurring equation

$$x^n - p\,x^{n-1} + q\,x^{n-2} - \&c. + q\,x^2 - p\,x + 1 = 0,$$

whose roots are a, b, c, &c.; prove that

$$\frac{a^2}{b^2} + \frac{b^2}{a^2} + \frac{a^2}{c^2} + \frac{c^2}{a^2} + \frac{b^2}{c^2} + \frac{c^2}{b^2} + \&c.$$

$$= (p^2 - 2q + \sqrt{n})(p^2 - 2q - \sqrt{n}).$$

15. A body projected in a given direction with a given velocity and attracted towards a given centre of force, has its velocity at every point : the velocity in a circle at the same distance :: 1 : $\sqrt{2}$; find the orbit described, the position of its apse, the magnitude of its axis and the law of force.

16. Prove that the image of a straight line formed by rays refracted through a sphere is a conic section; and find the magnitudes and positions of its axes and the latus rectum.

17. Find the sum of the series:

$$\frac{1}{1\,.\,2\,.\,3}+\frac{1}{2\,.\,3\,.\,4}+\frac{1}{3\,.\,4\,.\,5}+\text{\&c. } \textit{in infinitum.}$$

$$\text{and of } \frac{n\,x^3}{1\,.\,2\,.\,3}+\frac{n\,(n-1)\,x^4}{1\,.\,2\,.\,3\,.\,4}+\frac{n\,(n-1)\,(n-2)\,x^5}{1\,.\,2\,.\,3\,.\,4\,.\,5}$$

$$+\text{ \&c. to } n \text{ terms.}$$

18. Find that section of a sphere which attracts a corpuscle placed at a given point in the axis produced with the greatest possible force, the force of each particle $\propto \dfrac{1}{(\text{Dist})^2}$.

TUESDAY AFTERNOON.—Mr. King.

Third and Fourth Classes.

1. Insert 6 harmonic means between 1 and 20.

2. Find $\sin\theta$ from the equation

$$\sin 3\,\theta - 2\sin 2\,\theta + \sin^2\theta + 4\sin^3\theta = 0.$$

3. If A, B and C be the angles, and a, b and c the sides of a spherical triangle, and if $b + c = \pi$, prove that

$$\sin 2\,B + \sin 2\,C = 0.$$

4. How far must a given frustum of a sphere be immersed in a fluid, with its axis vertical, that the pressure on its two ends may be equal to n times the pressure on its curve surface?

5. An eye is situated in a given point of the straight line joining the centres of two given spheres, but not between them; find the visible surface of each.

6. If P_1 and P_2 be two sums, due respectively at the end of times t_1 and t_2, prove that the equated time of payment is expressed by

$$\frac{P_1 t_1 + P_2 t_2}{P_1 + P_2} - \frac{P_1 P_2}{(P_1 + P_2)^2} (t_1 - t_2)^2 r, \text{ very nearly,}$$

where r is the interest of 1£. for 1 year.

7. Having given the radius of a circle, find the area and perimeter of a regular octagon inscribed in it, and compare them with the area and perimeter of the circumscribing octagon.

8. Having given the contemporaneous altitudes of the Sun and a known Star, on a given day, and also the angular distance between them; find the latitude of the place and the hour of the day.

9. An uniform rod is made to vibrate about a point, so that the time of its oscillation is a minimum; find the force exerted on the point of suspension in any given position.

10. Trace the curve whose equation is

$$x^4 y^4 - a^4 y^4 + b^8 = 0, \text{ and find its area.}$$

11. Integrate the following differentials:

$$\frac{dx\sqrt{a^2 + x^2}}{x}, \quad \frac{(\sin x)^5 \, dx}{(\cos x)^3}.$$

$$(a^2 y + x^3)\, dx + (b^3 + a^2 x)\, dy = 0.$$

12. If a body describe an equilateral hyperbola, round a centre of force situated in the centre, and if θ be the angle described by the body from an apse in time t, prove that

$$\sin 2\theta = \frac{e^{4\sqrt{k}t} - 1}{e^{4\sqrt{k}t} + 1},$$

the force at distance 1 being represented by k.

TUESDAY EVENING.—Mr. King.

1. Find the least whole numbers which satisfy the equation

$$11x - 18y = 63.$$

2. Prove that $n(n-1)(n-2)$. &c. $(n-r)$ is divisible by $1.2.3$. &c. $(r+1)$, n being a whole number.

3. Determine the locus of a point so situated within a plane triangle, that the sum of the squares of the straight lines drawn from it to the angular points is constant; if the curve has a centre, determine its position.

4. Determine when the sum of the zenith distances of two known stars in a given latitude, is a maximum.

5. One end of a beam is connected to a horizontal plane by a hinge, about which the beam is suffered to revolve in a vertical plane; the other end is attached to a weight by means of a string passing over a pulley in the same vertical plane; find the position of equilibrium.

6. A parabola and hyperbola have the same vertex and the same axis; draw a tangent to the former which shall cut the latter in a given angle.

7. A cone containing a given quantity of fluid, has its axis inclined to the horizon at a given angle; find the time of emptying through a small orifice in its vertex.

8. Having given six straight lines, of which each is less than the sum of any two, determine how many tetrahedrons can be formed, of which these straight lines are the edges.

9. Two rods, of given lengths, are erected perpendicular to a given plane; find the locus of an eye in that plane, to which the sum of their apparent magnitudes will always be the same.

10. If A, B and C be the angles of a spherical triangle, a, b and c the opposite sides, and δ the distance of a point on the surface of the sphere, equally distant from the angular points; prove that

$$\tan^2\delta = \frac{\tan^2\frac{a}{2}\tan^2\frac{c}{2} - 2\tan\frac{a}{2}\tan\frac{c}{2}\cos B}{\sin^2 B}.$$

11. A semi-circle, the plane of which is vertical and base horizontal, has an uniform chain of given length, placed in a given position upon its circumference; find the velocity of the chain at the end of a given time.

12. If $ay^2 + bxy + cx^2 + dy + ex + f = 0$, be the equation to a curve of the second order; prove that the angles which its principal diameters make with the axis of x, may be determined from the equation

$$\tan 2\theta = -\frac{b}{a-c}.$$

13. It is required to graduate a horizontal dial, the style of which is in the meridian, and inclined to the horizon at a given angle, so that on a given day it shall shew the apparent time in a given latitude.

14. Find the centre of pressure of the sector of a circle, the axis of the sector being supposed to be vertical.

15. Find that point in the surface of a given paraboloid through which if two planes be drawn, one perpendicular and the other parallel to the axis, the sum of the areas of the sections shall be a maximum.

16. Apply the differential equations of motion to determine the density of the medium, so that a body may describe a given curve, about a given centre of force; and find the law of the density, when the curve is a circle and a force is situated in its circumference varying as $\frac{1}{D^n}$.

17. Two given weights are connected by a string passing through a hole in a horizontal plane; one of them is projected in any direction in the horizontal plane, the other descends vertically by the action of gravity; find the motion of the bodies, and the curve described on the plane.

18. If a body be projected in a medium, the resistance of which varies as the velocity, and be acted on by gravity, and another be projected *in vacuo* at the same angle, and with the same velocity, and acted upon by the same constant force, and if t_1 and t_2 be the times of describing two arcs in the medium, and *in vacuo*, so related to each other that the tangents at their extremities shall cut the axis at the same angle; then $e^{kt_1} - 1 = k t_2$; k being the resistance to velocity 1.

19. If a body describe an ellipse uniformly, round two centres of force situated in the foci; prove that the forces at any point of the ellipse are equal, and inversely proportional to the square of the corresponding conjugate diameter.

20. Find the locus of the vertices of all right-angled spherical triangles having the same hypothenuse; and from the equation obtained, prove that the locus is a circle when the radius of the sphere is infinite.

21. Three points move with equal velocities in three rectangular axes; one of them commences its motion from the origin, the other two from two given points equally distant from the origin; find the equation to the surface, to which the plane, passing through the contemporaneous positions of the points, shall always be a tangent.

22. Integrate the following differentials:

$$\frac{x^2\,dx}{(a+x)(x^2+b^2)},\quad \frac{dx}{x^5+2x^3+3x},$$

$$x^2 y\,dx - y^3\,dy = x^3\,dy,\quad x^3\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0,$$

$$z - px - qy = 3a\,p^{\frac{1}{3}}\,q^{\frac{1}{3}},$$

$$\text{where } p = \frac{dz}{dx} \text{ and } q = \frac{dz}{dy}.$$

23. Sum the following series:

$$\frac{1}{1.3.5} + \frac{2}{3.5.7} + \frac{3}{5.7.9} + \&c.$$

to n terms and to infinity:

$$\frac{1^2}{1^2-k^2}\cos\theta + \frac{2^2}{2^2-k^2}\cos 2\theta + \frac{3^2}{3^2-k^2}\cos 3\theta + \&c.$$

24. Integrate the following equation of differences:

$$u_{x+1} - a\,u_x = x^2.$$

1827.

MONDAY MORNING.—Mr. Maddy.

First and Second Classes.

1. There were five Sundays in February 1824; in what year will this happen again?

2. If C be the centre of an ellipse, and in the normal to
73 any point P, PQ be taken equal to the semi-conjugate at P, Q will trace out a circle round C.

3. A cord, the ends of which are joined, is suspended freely over two pegs in the same horizontal line, so as to form two catenaries, of which the arcs are $2s$ and $2s'$, and the tensions at the lowest points a and a'; prove that

$$s - s' : a' - a :: a' + a : s + s'.$$

4. Draw through a given point in the side of a spherical triangle, an arc of a great circle, cutting off a given part of the triangle.

5. If u be a homogeneous function of x, y, z, &c. of n dimensions, and p, q, r, &c. the values of $\dfrac{du}{dx}$, $\dfrac{du}{dy}$, &c.

$$(n-1)\,du = x\,dp + y\,dq + \&c.$$

6. The curve surface of a conical vessel of water, placed with its axis vertical and base uppermost, is composed of an infinite number of triangular sectors, admitting of revolution round hinges at the vertex of the cone, and confined by a string passing round the base; find the tension of the string.

7. Find when the inclination of the ecliptic to the horizon increases fastest.

8. If an equation has n equal roots, the equation formed by multiplying the terms by the terms of an arithmetical progression has $n-1$ of them.

9. A ray of light which passes through two media, bounded by parallel plane surfaces, will emerge parallel to its first direction, if the deviation in passing out of one medium into another under a given angle of incidence be supposed proportional to the difference of the densities of the media.

10. A body projected from a given point in a plane is attracted by forces $\frac{\alpha}{x^3}$ in the direction of x, and $\frac{\beta}{y^3}$ in the direction of y; prove that if the velocity and direction of projection be rightly assumed, it will describe a circle round the origin as a centre, and find how the velocity varies in different parts of the orbit.

11. There are two heaps of cards, the one of which contains three black and four red cards, and the other five black and two red; what is the probability that a person who takes up one card will draw a red one?

12. What probable and adequate cause has been assigned for the secondary planets always turning the same face towards their primaries?

13. If all possible ellipsoids be described of which the axes a, b, c are subject to the condition $c : a :: a : b :: b$: a given line; shew how the equation to the surface they all touch may be found, and find the equations of the curve in which it is touched by any one of the ellipsoids, and of the curve which is the locus of the intersections of all such curves. The centre and the directions of the axes being the same in all the ellipsoids.

MONDAY AFTERNOON.—Mr. Maddy.

Fifth and Sixth Classes.

1. Sum the series $15 + \frac{44}{3} + \frac{43}{3} + \&c. \ldots$ to 16 terms,

$$x^{\frac{5}{2}} - a\,x + \frac{a^2}{\sqrt{x}} - \&c. \ldots \text{ to } n \text{ terms},$$

and find the value of the recurring decimal 1.23434. . . . .

2. If a body be projected downwards with a given velocity, what is its velocity after describing a given space?

3. Find the differentials of $\log(\sin x)$ and $\frac{\sqrt[m]{(1+x^2)^2}}{\sqrt[n]{(1-x^2)^2}}$,

and the integrals of

$$\frac{d\,x}{\sqrt{a\,x - b\,x^2}}, \quad \frac{x^2\,d\,x}{x^2 - 1} \quad \text{and} \quad e^{\log x}\,d\,x.$$

4. Solve the equations

$$\left.\begin{aligned}(a^2)^x - (b^2)^y &= c\\ a^x - b^y &= d\end{aligned}\right\}; \quad x - 3 = \frac{3 + 4\sqrt{x}}{x}.$$

5. A body weighs four ounces *in vacuo*, and if another body which weighs three ounces in water be attached to it, the whole in water weighs two ounces and a quarter: find the specific gravity of the former body.

6. Compare the difference of the forces in the fixed and moveable orbits, with the force in a circle at the same distance described with the same angular velocity.

7. There are two air-pumps, one with a receiver A and barrel B, the other with a receiver B and barrel A; compare the quantities of air exhausted by them in t turns.

8. If a star whose right ascension is 19° 25′ pass over the meridian 2^h 18′ of sidereal time before the Sun, what is the Sun's right ascension when on the meridian?

9. Given the base, the vertical angle, and the difference of the sides of a plane triangle; find the remaining angles.

10. If $y = x^3 - 2x^2 + x + 4$, find the maximum and minimum values of y, distinguish them from each other, and shew that they are not the greatest and least values that y admits of.

11. Trace the curve whose equation is

$$y = \sqrt[3]{\frac{a^4}{x}} - x,$$

and find the area, and the solid formed by its revolution from $x = 0$ to $x = a$.

12. Expand $\sin x$ in powers of x, state whether in your result x is expressed in seconds or in parts of the radius, and convert it into the other.

13. Given the radius vector at any point of a parabola, and the angle it makes with the curve; find the latus rectum and the place of the vertex.

14. Find the pressure which a given power exerts by means of a common vice, the dimensions of which are given.

15. Extract the square roots of $7 + 4\sqrt{3}$ and $2\sqrt{-2} - 1$.

16. Prove that an object seen through the astronomical telescope appears inverted, but may be made to appear erect by two additional glasses, and find the magnifying power of the telescope thus formed.

17. Explain the Moon's phases, and why part of the disk is always visible.

18. To what depth will a given paraboloid placed with its axis vertical sink in a fluid of three times the specific gravity of itself?

MONDAY AFTERNOON.—Mr. Coddington.

Third and Fourth Classes.

1. According to what law must the centripetal force vary, that the areas *dato tempore* in all circles uniformly described about the centre may be equal?

2. What are the lines traced by the vertex and the focus of a parabola rolling on another equal to it, the vertices coinciding in one position?

3. At what distance from a luminous sphere must a point be situated so as to receive the greatest quantity of light from it?

4. How is the multiplication of two high numbers facilitated by a table of squares, or by one of cosines?

5. Prove that a spheroidal mirror may be made to reflect diverging rays accurately to one point: and by the help of this proposition find the common formula for a spherical mirror.

6. The times of the Sun's rising and setting being calculated for a certain place, what correction is necessary to make them serve for another place not far distant from it?

7. Prove, that when a mass entirely free is struck at any point, the motion of translation is the same as if the direction of the impact passed through the centre of gravity, and that of rotation as if the centre were fixed by an axis. By this proposition find the distance of the centre of percussion from a fixed axis.

8. Compare the volume of a sphere with that of the least cone that can be described about it.

9. The style of a horizontal dial being bent down, its edge coincides with the 9 o'clock hour-line. For what latitude was it constructed?

10. Supposing the Sun to remain above the horizon a given number of days, find the latitude.

11. Differentiate the continued fraction $\cfrac{x^2}{1 - \cfrac{x^2}{1 - \cfrac{x^2}{1 - \dots}}}$

12. Find, geometrically, the inclination of the path of a projectile to the horizon at a given time after the beginning of its motion, and prove by that means that the trajectory is a parabola.

MONDAY EVENING.—Mr. Coddington.

1. Explain the method of Geometrical Analysis, and by it solve the problem: In a given square to inscribe another square having its side equal to a given straight line. To what limitation is this line subject?

2. In how many ways can an equivalent for thirteen dollars, at three shillings, be given in English crowns and seven shilling pieces?

3. Shew that in general a parabola may be found which shall have a much more intimate contact with a given curve than any circle whatever.

4. If L be the length in miles of an arc of a great circle of the Earth, D the depression in feet of one extremity of it below a tangent drawn at the other, $D = \frac{2}{3} L^2$ nearly.

5. The voussoirs of a bridge being very small, and the equilibrium maintained by the vertical pressure of the masonry above them, of what portion of a circle must the intrados consist, that the ascent of the bridge from a level road may be continuous, supposing the thickness of the arch at its summit to be to the radius of the intrados as 1 to $5\frac{1}{3}$?

6. Prove that Brinkley's formula for the mean refraction is reducible to the same form as Bradley's.

7. Shew that Saturn's ring cannot be a homogeneous and regular *solid* of revolution.

8. Given the position of the Moon's nodes, and the inclination of her orbit to the ecliptic, to find when her latitude and declination are equal.

9. An uniform elastic string being of such a length that when it hangs vertically if an equal quantity were appended to the lowest point it would stretch it to twice that length, what weight must be appended at the middle point that the increase of length may be three quarters of the original?

10. In a chart on Mercator's projection the length of the meridian from the radius of 30° to that of 60° is to the radius of the sphere as the natural logarithm of $\frac{\sqrt{3}+1}{3-\sqrt{3}}$ to 1.

11. Show that with any four lines, each of which is less than the sum of the others, it is possible to construct a trapezium which may be inscribed in a circle.

12. How may the square of 50973 be found approximately by a table of squares of whole numbers from 1 to 1000?

13. If AB be the diameter of a circle, C the centre, FAG a tangent at A, ACF one third of a right angle, and FG triple of the radius, BG being joined will be very nearly equal to the half circumference.

14. Compare the probabilities of taking an odd or an even number of balls from a heap containing a given number.

15. How must the equidistant seats of a lecture-room be constructed, so that persons of equal height sitting on them, and directing their eyes to the same given point may be equally able to see over each others' heads?

16. Find the disturbing forces of Venus on the Earth when their heliocentric longitudes differ by 45°.

17. If the refraction of light be the effect of any causes which act throughout a certain distance from the surface of a

medium, and the intensity of which depends solely on the distance from that surface, the ratio of the sines of incidence and refraction must be constant.

18. Supposing the latitude of a star to be 60°, its longitude 95°, and that of the Sun 65°, what is the aberration in longitude? In what sense is $20''.25$ the maximum of aberration?

19. In a stereographic projection of the sphere it is required to draw a great circle through two given points.

20. Explain the manner of using Gunter's logarithmic scales.

21. Solve the equation $\phi\, x^2 = \phi\,(2\,x) + 2$.

22. Find two whole numbers of which the product shall be divisible by the sum.

23. Given $z = x + e^z$. Required z in terms of x.

24. Explain the division of a string which produces the several notes of the diatonic scale. What alteration would be made in the general pitch by assuming 256 instead of 240 for the number of vibrations constituting the tenor C?

TUESDAY MORNING.—Mr. Coddington.

First and Second Classes.

1. Explain the method of finding the refracting power of a soft substance by placing it between glass lenses.

2. Of all conical surfaces of equal altitudes, determine that which exerts the greatest attraction on a particle at its vertex.

3. A circle being described on the axis major of an ellipse, and a tangent drawn to each curve at the points where an ordinate to the axis meets them, find where the angle between these tangents is greatest: and shew what is the ultimate point of contact in this case, when the eccentricity of the ellipse is diminished *sine limite*.

4. Prove the rule for multiplying decimals together without any reference to vulgar fractions.

5. If two bodies S and P attracting each other with forces varying inversely as the square of the distance, revolve about each other, S being much greater than P, the actual time of one revolution will be less than if S were immoveable in the ratio of 1 to $1 + \frac{P}{2S}$.

6. On the supposition of a homogeneous atmosphere, the refraction may be expressed by the formula

$$r = \frac{m-1}{\sin 1''} \tan\left(Z - \frac{\delta}{(m-1)(1+\delta)} r\right),$$

δ being the ratio of the height of the homogeneous atmosphere to the radius of the Earth.

7. Find the equation to a conical surface in general, and deduce from it that of a common right cone.

8. In a plane triangle ABC, when the side b is much less than a, the angle B may be found by the formula

$$B = \frac{b}{a}\,\frac{\sin C}{\sin 1''} + \frac{b^2}{a^2}\,\frac{\sin 2C}{\sin 2''} + \frac{b^3}{a^3}\,\frac{\sin 3C}{\sin 3''} - \ldots.$$

9. What curve is that in which the perpendicular from the origin on the tangent is always equal to the abscissa?

10. Find the value of the fraction $\frac{3.5.9.17\ldots.}{2.4.8.16\ldots.}$ when its numerator and denominator are continued *sine limite*.

11. If S be half the sum of the sides of a spherical triangle, D its area,

$$\tan\frac{D}{4} = \sqrt{\left\{\tan\frac{S}{2}\ \tan\frac{S-a}{2}\ \tan\frac{S-b}{2}\ \tan\frac{S-c}{2}\right\}}.$$

12. Supposing the orbit of a Comet to lie in one plane, if the force attracting it towards the Sun vary as that power of the distance whose index is $2-\delta$, (δ being a small fraction), the heliocentric angle between two successive perihelia will be $360° \frac{\delta}{1+\delta}$.

TUESDAY AFTERNOON.—Mr. Coddington.

Fifth and Sixth Classes.

1. Prove geometrically that if A, B be any two arcs, rad $\times \cos (A - B) = \cos A \cos B + \sin A \sin B$.

2. The length of a floor being 10 feet 6 inches, and the breadth 9 feet 3 inches, find the area by duodecimal multiplication, and define the several terms of the product.

3. The second and third terms of a geometrical progression are together equal to 24, and the two next to 216: what is the first?

4. Investigate the rule of Alligation, in which the prices of the ingredients and of the mixture are given, to find the proportions of the former.

5. Given a, b, c the sides of a plane triangle, find the radius of the inscribed circle.

6. Draw a straight line touching a circle at a given point, without any other instruments besides a parallel ruler and a pencil.

7. By what modification may formulæ for spherical triangles be adapted to plane ones?

8. If n be any prime number greater than 3, $n^2 - 1$ is divisible by 12.

9. Given the altitudes of two known stars at the same instant of time: required the latitude of the place. How may this problem be solved geometrically on a sphere?

10. If two elastic balls in the ratio of 1 to 3 meet directly with equal velocities, the larger one will remain at rest.

11. Continue in both directions the harmonic progression of which 4 and 6 are adjacent terms.

12. Prove geometrically that $\frac{1}{2}(y\,d\,x - x\,d\,y)$ is the element of a sectorial area about the origin of the co-ordinates.

13. Compare the force at a given point of an ellipse described about the focus, with that in a circle at the same distance described with the velocity in the ellipse at that point.

14. Two given weights being attached to given points in the circumference of a wheel, find the position in which the greatest weight will be supported on the axle.

15. Prove the following formula for small arcs,

$$l \sin x = l\,x + \tfrac{1}{3}\,l \cos x.$$

16. A pair of conjugate hyperbolas being given, find their centre.

17. If $\dfrac{d\,x}{1 + x^2} + \dfrac{d\,y}{1 + y^2} = 0$, find an algebraical value of y in terms of x.

TUESDAY AFTERNOON.—Mr. Maddy.

Third and Fourth Classes.

1. Convert $17° \; 25' \; 8''$ into time at the rate of $15°$ to one hour.

2. If $C_0 = \cos a \cos b \cos c \ldots.$, $C_n =$ the sums of the products of all the cosines but n, multiplied by the sines of those n,

$$\cos(a + b + c + \&c.) = C_0 - C_2 + C_4 - \&c.$$
$$\sin(a + b + c + \&c.) = C_1 - C_3 + C_5 - \&c.$$

3. Find at what angle a plane must be inclined to the side of a cone in order that the section may be a rectangular hyperbola: and determine the least vertical angle of the cone for which the problem is possible.

4. Shew that a tennis ball projected along an inclined roof, but not in the direction in which it would naturally fall, describes a parabola, and find its latus rectum, having given the inclination of the roof, and the velocity and direction of projection.

5. In the ordinate PN of a parabola πN is taken proportional to the curvature of the parabola at P; find the area of the curve which is the locus of π.

6. If $\tan\theta$ be assumed $= \dfrac{b}{a}$,

$$\sqrt[n]{a \pm b\sqrt{-1}} = (a^2 + b^2)^{\frac{1}{2n}} \left\{\cos\frac{\theta}{n} \pm \sin\frac{\theta}{n}\sqrt{-1}\right\}.$$

7. Find where the space due externally to the velocity in an ellipse, force in focus, is thrice the space due internally.

8. Trace the curve whose equation is $a^{\frac{3}{2}}\, y = (x - a)^2 \sqrt{x}$.

9. What must be the form of a surface of revolution in which, when filled with water which runs out by a small orifice at the lowest point, the surface descends from its greatest altitude with an uniformly accelerated motion?

10. A small pencil of rays, parallel to the axis of a hemisphere of denser medium, is incident nearly perpendicularly on the convex surface; find the geometrical focus of emergent rays.

11. If ordinates $y_1, y_2 \ldots y_n$ be drawn, at equal intervals beginning from the origin, to the catenary whose equation is

$$\frac{y}{a} = \tfrac{1}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right);$$

prove that

$$(y_1)^n = \left(\frac{a}{2}\right)^{n-1} \left\{y_n + n\,y_{n-2} + \frac{n(n-1)}{1\,.\,2}\,y_{n-4} + \&c.\right\},$$

and write down the last term of the series.

12. Find the azimuth of two known stars which are seen at the same instant in one vertical plane.

13. If a cylinder (weight w), attached by a string passing over a pulley to a weight $= \frac{1}{2} w$, be just immersed vertically in a fluid of the same specific gravity as itself; find the greatest velocity acquired by the cylinder, and the time of its ascending to its greatest height.

14. Explain clearly, from elementary principles, why the Moon's attraction causes a tide on the opposite side of the Earth.

TUESDAY EVENING.—Mr. Maddy.

1. What part of its bulk at 60° does a body expand for each additional degree of temperature, supposing it to expand .05 parts of the magnitude which is at 32° for each degree above 32°?

2. If A, B, C be the angles, a, b, c the sides of a plane triangle,

$$\sin (A - B) : \sin C :: a^2 - b^2 : c^2.$$

3. Find the part of a sphere cut out by three planes passing through its centre, and inclined to each other at angles of 120°.

4. Determine the points of a given ellipse in which the sum of the conjugate diameters is the greatest or least possible, and distinguish the maximum from the minimum.

5. If a circle of given radius oscillate flatways through a small angle, determine the content of the solid which it traces out, having given the time of the oscillation and the whole angle through which it oscillates.

6. At points of a curve where the curvature is a maximum or a minimum the circle of curvature has a contact of a higher order than the second.

7. Find the equation to the curve cutting at right angles all *equal* parabolas having their axes in the same line.

8. Prove that the image of a straight line, seen through a prism, the angle of which is small, is a straight line; and compare the angle between the object and image with the angle of the prism when the object is parallel to one side of the prism, and in a plane which is perpendicular to the two sides.

9. Draw all the rectilineal asymptotes to the curve whose equation is $y = \dfrac{x^3 + a\,x^2 + a^3}{x^2 - a^2}$; and trace the curve whose equation is $x^4 + y^4 = 2\,a\,x\,y^2$.

10. A known circumpolar star reaches its maximum azimuth at two different places at the same instant; having given the values of the maximum azimuth at the two places, find their latitudes and the difference of longitude.

11. Find the density of the air and the altitude of the mercury in a barometer at a given depth within the Earth; gravity being supposed to vary as the distance from the Earth's centre, and the temperature of the air, from the surface to where the barometer stands, to remain constant.

12. If the force vary inversely as the 7th power of the distance, and a body be projected from an apse with a velocity which is to the velocity in a circle at the same distance :: 1 : $\sqrt{3}$; find the polar equation to the curve described, and transform it to rectangular co-ordinates.

13. Explain what is meant by the polarization of light, and prove experimentally that it may be produced by reflexion from transparent media.

14. Find the equation to the curve cutting off equal arcs from all circles which have their centres in the same line, and their circumferences passing through a given point in that line;

and prove that the distances from this point at which the curve cuts the line are as the numbers $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}$, &c. and the tangents of the angles at which it cuts it as 1, 3, 5, &c.

15. Force varying inversely as the square of the distance, if a body be projected with n times the velocity in a circle at the same distance, and in a direction making an angle α with the distance; the angle θ between the axis major and the distance may be determined from the equation

$$\tan(\theta - \alpha) = (1 \backsim n^2) \tan \alpha.$$

16. A cone of given specific gravity rests in a given fluid with its vertex immersed and axis vertical, shew that the nature of the equilibrium will not be affected by altering the altitude of the cone; and find the vertical angle when the equilibrium is indifferent.

17. A plane passing through a given point, and always touching a surface of the second order, traces out a plane curve on the surface.

18. Find how long a given sphere, suspended by a twisted string which is suffered to untwist, will continue to turn in the same direction.

19. If two given equal weights sustain each other by a string passing over a smooth curve, the plane of which is vertical, the sum of the pressures on any arc depends only on the directions of its extremities.

20. Integrate the differentials and differential equations

$$\frac{x \log x \, dx}{\sqrt{1 - x^2}}, \quad \frac{dx}{\sqrt[3]{1+x} - \sqrt{1+x}}, \quad \frac{d^n y}{dx^n} = y,$$

$$dx + 2x\,dy = x^2 y^2\,dy.$$

21. Sum the series $\cos x - \cos 2x + \cos 3x -$ &c. *ad infinitum*;

$$\frac{1}{2 \cdot 3 \cdot 5} + \frac{2}{3 \cdot 5 \cdot 9} + \frac{4}{5 \cdot 9 \cdot 17} + \frac{8}{9 \cdot 17 \cdot 33} + \text{\&c.}$$

to n terms; and find the integral of $a^x (1 + x)^2$.

22. If x and y be co-ordinates of any point of the shortest line drawn between two given points on a surface formed by the revolution of a plane curve round the axis of z, and ds the differential of the line, prove that $x\,dy - y\,dx = c\,ds$, c being a constant; and find the equations to the line when the surface is a paraboloid.

23. If the particles of a hollow elastic cylinder be so arranged that on its being subjected to a given internal pressure they may all be in the same given degree of dilatation, find how the thickness must be altered, in order that the strength of the cylinder may increase in arithmetical progression; the internal radius of the cylinder being supposed to remain constant.

24. Prove that there are generally either two homogeneous fluid spheroids of equilibrium or none, for the same time of rotation; and supposing the eccentricity of the one spheroid very small, find the ratio of the axes in the other.

1828.

MODERATORS,

MR. WHEWELL, Trinity. MR. KING, Queen's.

EXAMINERS,

MR. MARTIN, Trinity. MR. MELVILL, St. Peter's.

FRIDAY MORNING.

QUESTIONS IN PURE MATHEMATICS.

First, Second, Third and Fourth Classes.

1. IN a given circle to inscribe an equilateral and equiangular pentagon.

2. If four quantities of the same kind be proportional, the greatest and least together are greater than the other two.

3. What is the amount of 37cwt. 2qrs. 14lbs., at £7. 10*s*. 9*d*. per cwt.?

4. Find $\sqrt{3\sqrt{3}+2\sqrt{6}}$, in the form of a binomial surd.

5. Find an expression for the sum of a decreasing geometric series, and explain clearly the possibility of an infinite number of terms having a finite sum.

6. Find the value of an annuity of £100. to commence 10 years hence and to continue for ever, allowing compound interest.

7. Determine the number of permutations of n things taken all together, supposing the same quantities to recur.

8. Prove the binomial theorem when the index is fractional. Apply it to expand $(a^2 - \frac{2}{3} x^2)^{\frac{3}{2}}$ to five terms.

9. Given the sines and cosines of two arcs, find the sine of their sum and difference.

10. In the equation $x^4 + 8x^3 + x^2 - x - 10 = 0$ take away the second term, and then find the reducing cubic.

11. Find the sum of the sixth powers of the roots of the equation

$$x^3 - x - 1 = 0.$$

12. Shew that *Cardan's* solution applies only to those cases in which the equation has two impossible roots, unless two of the roots be equal.

13. Transform the continued fraction $\cfrac{1}{a + \cfrac{1}{b + \cfrac{1}{c + \&c.}}}$ into a series of converging fractions, and prove that each of the latter approaches the value of the original fraction more nearly than the preceding.

14. If two chords of a parabola move parallel to themselves intersecting each other, the rectangles of their segments are in a constant ratio.

15. In the ellipse all the circumscribing parallelograms are equal.

16. Define the radius of curvature and prove that in an ellipse it $= \dfrac{CD^2}{PF}$.

17. Find the algebraic equation to the cissoid of *Diocles*, trace the curve, and deduce the polar equation, the cusp being the pole.

18. In a spherical triangle the sines of the angles are as the sines of the opposite sides.

19. Prove without the use of the integral calculus that the solid content of a cone is one-third that of a cylinder of the same base and altitude.

20. What is the logarithm of any number? Why is the common system selected? What is its base? Explain and prove the rule for proportional parts.

FRIDAY AFTERNOON.

QUESTIONS IN NATURAL PHILOSOPHY.

First, Second, Third and Fourth Classes.

1. If a point be kept at rest by three forces acting upon it at the same time, any three lines which are in the direction of those forces and form a triangle will represent them.

2. A body is placed on a horizontal plane; find when it will be supported.

3. By what experiments is the third law of motion established?

4. Two bodies of given magnitudes and elasticity, impinge directly upon each other with given velocities; find the velocity of each after impact.

5. A body is projected from a given point in a given direction with a given velocity, and acted upon by gravity; find where it will strike a given plane.

6. How is it shewn that fluids press equally in all directions? Apply this principle to the explanation of the Hydrostatical Paradox and *Bramah's* Press.

7. Find the density of the air in a common condenser after t descents of the piston.

8. When different planes move in directions perpendicular to their surfaces in different fluids and with different velocities, the resistances will be as the squares of their velocities $\times$ densities of the fluids $\times$ areas of the planes.

9. If parallel rays be incident nearly perpendicularly upon a spherical refracting surface, the distance of the geometrical focus of refracted rays from the surface, is to its distance from the centre, as the sine of incidence to the sine of refraction.

10. Prove that the image of a straight line formed by a plane refracting surface is a straight line, and having given the inclination of the line to the refracting surface, find the inclination of the image.

11. Construct the solar microscope.

12. Construct *Gregory's* telescope, find its magnifying power and its greatest field of view.

13. Define continuous curvature, and shew that the arc, chord, and tangent of any curve of continuous curvature are ultimately equal.

14. Enunciate and prove *Newton's* eleventh Lemma.

15. If equal areas be described by a body in equal times about a given point in a given plane, the body is urged by a force tending to that point. (Newt. Book I. Prop. 2.)

16. If a body be projected from a given point in a given direction with a given velocity about a centre of force varying as the distance, shew that it will describe an ellipse having its centre in the centre of force, and find the magnitudes and positions of the axes. (Newt. Book I. Prop. 10. Cor. 1.)

17. Find the law of the force tending to the focus of an hyperbola. (Newt. Book I. Prop. 12.)

18. State the principal arguments for the diurnal rotation of the Earth round an axis, and its annual motion round the Sun.

19. Explain the theory of the aberration of light, and define clearly the plane in which it takes place.

20. Having given the right ascension and declination of a star; find its latitude and longitude, and adapt the formulæ to logarithmic computation.

EVENING PROBLEMS. MR. WHEWELL.

1. The sums of the co-efficients of the even and odd terms of any power of $a+b$ are equal.

2. If the side of a pentagon inscribed in a circle be 1, the radius is

$$\frac{\sqrt{5+\sqrt{5}}}{\sqrt{10}}.$$

Prove this, and hence find the sine of 36° to five places of decimals.

3. A box is full of small spherical shot: what portion of the space is empty?

4. A ladder of uniform thickness rests with its lower end on a horizontal plane, and its upper end on a slope inclined 60° to the horizon. The ladder makes an angle of 30° with the horizon; find the force which must act horizontally at the foot to prevent sliding.

5. Find the velocity and direction of projection of a ball, that it may be 100 feet above the ground at one mile distance and may strike the ground at three miles.

6. If a right cone of which the semi-angle is γ be cut by a plane making an angle δ with its axis, the ellipse thus obtained will have its minor-axis : major-axis :: $\sqrt{\sin(\delta+\gamma)\sin(\delta-\gamma)}$: $\cos\gamma$.

7. If a line be drawn through the focus of an ellipse making an angle θ with the major-axis, and tangents be drawn at the extremities of this line: these tangents will make an angle ϕ, such that $\tan\phi = \dfrac{2e\sin\theta}{1-e^2}$.

8. What must be the relation of the distances from the Sun, of a superior and inferior planet, that their synodical revolutions may be equal?

9. A telescope consists of three convex lenses whose focal lengths are $30m$, $3m$, m, the two latter being at a distance $2\ m$. Find its magnifying power, and the distance of the first two lenses. Trace the course of the rays.

10. Find the n quantities $x_1, x_2, x_3, \ldots x_n$, from the n equations:

$$\begin{aligned}
x_1 \quad + x_2 \ldots\ldots + x_n \quad &= A,\\
a_1 x_1 + a_2 x_2 \ \ldots . + a_n\, x_n \quad &= 0,\\
a_1^2 x_1 + a_2^2 x_2 \ldots . + a_n^2 x_n \quad &= 0,\\
\ldots\ldots\ldots\ldots\ldots\ldots &\ldots ;\\
a_1^{n-1} x_1 + a_2^{n-1} x_2 \ldots + a_n^{n-1} x_n &= 0 ;
\end{aligned}$$

and obtain symmetrical expressions for x_1, x_2, &c.

11. Shew that

$$\frac{1}{(a+x)^m} = (p+1) \text{ terms of the expansion of } (a+x)^{-m}$$

$$+ \frac{x^{p+1}}{(a+x)^m} \times m \text{ terms of the expansion of } (\overline{a+x} - a)^{-\overline{p+1}}.$$

12. Draw the curve whose equation is $y = \sin x + 2 \sin 2x$. Find all its points of maximum, flexure and intersection ; and shew after what values of x its form will recur.

13. Integrate

$$x^3 dx \sqrt{\frac{1+x^2}{1-x^2}}; \quad x^{nr} dx \begin{Bmatrix} x = 0 \\ x = 1 \end{Bmatrix};$$

$$\frac{d^2 x}{dt^2} = 2x - 5y, \ \frac{d^2 y}{dt^2} = x + 2y;$$

and sum the series

$$1 + \frac{1}{2} x + \frac{1.3}{2.3} x^2 + \frac{1.3.5}{2.3.4} x^3 + \&\text{c. } \textit{in infinitum}.$$

14. In a given sphere rests a given plane triangle of uniform thickness ; find the angle which it makes with the horizon.

15. Find the form of a uniform chain suspended from any two points on the surface of an upright cone, and resting on the curve surface. Find the tension when it becomes a horizontal circle.

16. On a given triangle a pyramid is to be constituted of a given content. Determine it so that its surface may be the least possible.

17. *A* throws 6 dice, *B* throws 12, *C* throws 18. Compare the chances of *A* throwing one six, *B* two sixes and *C* three sixes.

18. Three stars *A*, *B*, *C* are very nearly in a great circle, the angle made by *A* and *C* at *B* being $180-\beta$, where β is small. Shew that if t be the time which elapses between AB and BC being vertical,

$$t = \frac{\sin z}{\cos a \cos l} \, \frac{\beta}{15},$$

where z is the zenith distance and a the azimuth of B, and l the latitude of the place.

19. A style projects from the vertex of an upright cone; trace the hour lines on the surface of the cone; and find the time in each day during which the dial will serve.

20. In a steam engine working *expansively*, the influx of steam is stopped when it has filled $\frac{1}{m}$ of the cylinder, and the piston is afterwards driven by the expansion of the steam. Compare the effect of a *given quantity* of steam so employed with its effect when it is not stopped; the effect being measured by the force $\times$ space moved through.

21. A straight rod moves on a smooth horizontal plane, subject to the condition of always passing through a given point: determine its motion. Prove that the varying centre of gyration of the rod with respect to the fixed point will describe areas uniformly about that point.

22. If a body fall to the Earth in the time t'', the deviation *to the east* of the point from which it fell will be $\frac{1}{3} g \alpha t^3 \cos l$;

where l is the latitude, and a the angle described by the Earth in 1″.

23. If the mass of the Earth increase slowly and uniformly, find the resulting *equation* of the Moon's place at any given time, the orbit being nearly circular.

24. Explain briefly the optical experiments and theories to which the following terms refer: *Fits of easy transmission and reflection; Polarization; Plane of Polarization; Depolarization; Depolarizing Axis; Fringes; Interferences.*

25. Mention the steps of the proof by which *Newton* shewed that every particle of matter gravitates to every other particle with a force which is inversely as the square of the distance.

SATURDAY MORNING.

QUESTIONS IN PURE MATHEMATICS.

First, Second, Third and Fourth Classes.

1. Similar triangles are to one another in the duplicate ratio of their homologous sides.

2. If a straight line be at right angles to a plane, every plane passing through that straight line is at right angles to the same plane.

3. If a, b, c be the sides of a plane triangle, and A the angle opposite to a, and $S = \frac{a+b+c}{2}$, prove the four following formulæ:

$$\sin A = \frac{2}{bc} \sqrt{S(S-a)(S-b)(S-c)};$$

$$\sin \frac{A}{2} = \sqrt{\frac{(S-b)(S-c)}{bc}};$$

$$\cos\frac{A}{2} = \sqrt{\frac{S\ (S-a)}{b\,c}}\,;$$

$$\tan\frac{A}{2} = \sqrt{\frac{(S-b)\,(S-c)}{S\ (S-a)}}\,.$$

State under what circumstances each of these formulæ may be used with the greatest advantage.

4. Sum the following series:

$$\left.\begin{array}{l} \cos a + \cos 3\,a \quad + \ \cos 5\,a \quad + \ldots . \\ \text{and } \tan a + 2\tan 2\,a + 2^2 \tan 2^2 a + \ldots . \end{array}\right\} \text{ to } n \text{ terms.}$$

5. If A, B, C be the angles of any spherical triangle, and a the side opposite to A, prove that

$$\cos a = \frac{\cos A + \cos B\ \cos C}{\sin B\ \sin C}.$$

6. The sum of the squares of any two conjugate diameters in an ellipse is constant.

7. Investigate the rule for determining the greatest common measure of any two quantities, and apply it to find the greatest common measure of

$$x^3 - 11\,x^2 + 39\,x - 45 \text{ and } 3\,x^2 - 22\,x + 39.$$

8. The co-efficient of the second term of an equation with its proper sign, is the sum of the roots with their signs changed; the co-efficient of the third term is the sum of the products of every two roots with their signs changed; the co-efficient of the fourth term is the sum of the products of every three roots with their signs changed; &c. &c.

9. Solve the equation $x^3 - 6\,x = 4$, by Trigonometry, and obtain a numerical result.

10. In how many years will a sum of money treble itself at $4\frac{1}{2}$ per cent. compound interest?

$$\log 3 = .4771213,$$

$$\log 1.045 = .0191163.$$

11. Find the least whole positive numbers which will satisfy the equation $7x - 9y = 29$.

12. Assuming the expansion of a^x, deduce a converging series for the logarithm of any number; and apply it to compute the *Napierian* logarithm of 5 to seven places of decimals, that of 2 being .6931471; and shew how these two logarithms determine the modulus of *Briggs'* system.

13. Define the differential co-efficient of any function, and from that definition find the differential co-efficient of $\frac{a^2 + x^2}{a - x}$, and of $\tan x$.

14. Investigate *Maclaurin's* theorem independently of *Taylor's* theorem, and apply it to find the series for a circular arc in terms of its sine.

15. If y be a function of x, determine the conditions requisite for y to be a maximum or minimum; and exemplify the theory when $y = (x - a)^n$, both when n is even, and when it is odd.

16. Shew how to determine when a curve is concave, and when convex to the axis. Trace the curve whose equation is

$$a^3 y = x^4 - b x^3 - b^2 x^2,$$

and determine the number and nature of its singular points.

17. Find the differential of the arc of any curve, and apply it to determine the length of the common parabola.

18. Explain the method of resolving any rational fraction into its simple or quadratic factors, and shew the use of such resolution in the integration of

$$\frac{dx}{(x^3 - x^2)(x^2 + x + 1)}.$$

SATURDAY AFTERNOON.

QUESTIONS IN NATURAL PHILOSOPHY.

First, Second, Third and Fourth Classes.

1. The equilibrium of the screw will take place when the power is to the weight as the distance of two contiguous threads to the whole circle described by the point where the force is applied.

2. Find the time of a body's descent down any arc of a cycloid, and shew that the times of the whole oscillations are as the square roots of the lengths of the strings.

3. If any number of forces act in the same plane upon a rigid body, determine their resultant, and the equation of the straight line in which the resultant acts.

4. Prove the formula for the place of the centre of gravity of any body, viz. $h = \frac{\int x\, d\, m}{m}$, and apply it to find the centre of gravity of a common parabola.

5. If a rigid body oscillate about a horizontal axis, find the length of a simple pendulum which shall oscillate in the same time.

6. A body falls towards a centre of force which varies as some power of the distance, determine the cases in which we can integrate so as to find the time of descent.

7. Knowing the force, which varies as $\frac{1}{D^2}$, and the velocity of projection from a given point, to find the path described. (Newton, Book I. Prop. 17.)

8. State and prove *Newton's* construction for the path of a body projected from an apse with a velocity less than that acquired by falling from an infinite distance, and acted upon

by a force varying inversely as the cube of the distance. (Newton, Book I. Prop. 41. Cor. 3.)

9. If the force vary as $\frac{1}{A^2} + \frac{1}{A^3}$, find the angle between the apsides by *Newton's* method. (Prop. 45.)

10. Describe the variations which take place in the inclination of P's orbit during one revolution of the line of Nodes. (Newton, Book I. Prop. 66. Cor. 10.)

11. When a ray of light passes through a prism in a plane perpendicular to its axis, the deviation is a minimum when the incident and emergent rays make equal angles with the sides.

12. Find the field of view in *Galileo's* Telescope.

13. Find the longitudinal aberration of parallel rays refracted by a spherical surface.

14. The height of a homogeneous atmosphere is the same for whatever distance above the Earth's surface we find it.

15. Compare the resistance on the arc of a plane curve moving in a fluid in the direction of its axis, with the resistance on the base; and apply the formula to the case of a semi-circle.

16. Given three altitudes of a known star observed very near the meridian, and the differences of the times of observation; determine the latitude of the place.

17. Describe the manner in which *Bradley* discovered Aberration, and in which he distinguished Nutation from it.

18. If z be the true zenith distance, P the horizontal parallax, p the parallax in seconds,

$$p = \sin P \frac{\sin z}{\sin 1''} + \sin^2 P \frac{\sin 2z}{\sin 2''} + \sin^3 P \frac{\sin 3z}{\sin 3''} + \&c.$$

EVENING PROBLEMS.—Mr. King.

1. If to the square of any number not divisible by 3 the number 2 be added, the result is divisible by 3.

2. Reduce $a\sqrt[4]{-1}+b\sqrt[6]{-1}$ to the form of $\alpha+\beta\sqrt{-1}$.

3. Two straight lines are inclined to each other at a given angle, find the area of all the circles which can be described touching each other and the two given lines, the position of the centre of the last circle being given.

4. Find the m^{th} differential co-efficient of $\sqrt{\cos x}$.

5. Find that point in the surface of a spherical triangle from which if straight lines be drawn to the angular points the pyramid thus formed shall be a maximum.

6. Find the equation to the curve from any point of which if two tangents be drawn to a given ellipse, the angle contained between them shall be constant.

7. Having given the latitude of the place, find the day of the year on which the shadow of a given ellipse placed perpendicular to the meridian with its major axis vertical will be an ellipse of half the eccentricity.

8. The interval between the passages of a known circumpolar star through the plane of a vertical instrument with a given azimuth is observed; find the latitude of the place and the area of the spherical surface contained between the vertical circle and the apparent path of the star.

9. A uniform rod is at liberty to move freely in a vertical plane about a horizontal axis; find the nature of the circumference of a wheel which revolving uniformly about a given horizontal axis shall cause the rod to revolve uniformly also: the point of contact of the wheel and the rod being always at the same distance from the point of suspension.

10. Perpendiculars are drawn from a given point upon an infinite number of planes all passing through another given point; find the locus of the intersections of the perpendiculars with the planes.

11. Apply the differential expression for the volume of any solid referred to three rectangular co-ordinates to find the volume of a portion of a paraboloid whose equation is

$$x^2 + y^2 = 2az,$$

cut off by a plane whose equation is

$$By - Cz = 0.$$

12. A heavy piston descends by its own weight in a close cylinder filled with atmospheric air; find the velocity at any point of its descent and shew how to approximate to the whole length of the oscillation.

13. The particles of a fluid mass are attracted to two equal constant centres of force and a uniform motion of rotation is given to the mass about the line joining those centres; find the equation to the surface which the fluid will assume.

14. A hexagonal pyramid whose sides are isosceles triangles is placed with its base on the plane of x and y; find the sum of the projections of its sides on the three co-ordinate planes.

15. A ring slides down a perfectly smooth rod revolving uniformly in a vertical plane; find the motion of the ring.

16. An oblique parallelopiped oscillates about one of its edges which is in a horizontal position; determine its motion and the pressure it exerts against the axis of suspension in any position.

17. If an ellipsoid, whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, be cut by a plane passing through the origin perpendicular to the plane of xy, prove that the normals to the surface drawn from every point of the intersection of the plane with the ellipsoid will cut the plane of xy in a straight line, and find the equation to that line.

18. Explain the use of observations made by reflexion on the Polar Star in adjusting a transit instrument.

19. Prove that $1 \cdot 2 \cdot 3 \,.\, \&c. \; x = \sqrt{2 \pi x} \left(\frac{x}{e}\right)^x$ very nearly, when x is large, and shew the utility of this formula in the solution of the following problem.

20. In a pack of 52 cards, containing an equal number of red and black cards, determine the probability that in drawing any even number of cards, there shall be an equal number of red and black cards: supposing that the probability of drawing any even number is the same. A numerical result is required.

21. Solve the following equation of differences,

$$u_{x+2} + 2 a u_{x+1} + a^2 u_x = A x^3 + B x^2 + C x.$$

22. A particle is placed any-where within a triangle, the sides of which are composed of particles attracting with forces varying as $\frac{1}{D^2}$; find the direction in which it will begin to move.

23. If a body oscillate in a cycloid, in a medium the resistance of which varies as the velocity, and s be the first arc of descent, prove that the whole space described by the body before the motion ceases

$$= s \,.\, \frac{e^{\frac{k\pi}{\sqrt{\left(\frac{g}{4a} - k^2\right)}}} + 1}{e^{\frac{k\pi}{\sqrt{\left(\frac{g}{4a} - k^2\right)}}} - 1},$$

where $2k =$ resistance to velocity 1, $g =$ gravity, $2a =$ diameter of the generating circle, and $\pi =$ the semi-circumference of a circle the radius of which is 1.

24. Shew how very small secular inequalities in the mean motions of two planets may be introduced when their mean motions are nearly commensurable.

MORNING PROBLEMS.—Mr. Whewell and Mr. King.

1. If n be greater than 3, shew that $\sqrt{n} > \sqrt[3]{n+1}$.

2. If any two circles, the centres of which are given, intersect each other, the greatest line which can be drawn through either point of intersection and terminated by the circles is independent of the diameters of the circles.

3. If a and b be the semi-axes of an ellipse, and θ and ϕ the angles which any two conjugates make with the major axis, prove that $\tan\theta\tan\phi = \frac{b^2}{a^2}$.

4. Sum the series

$$\left.\begin{array}{l}\frac{1}{1.3}\,\frac{1}{2^3} - \frac{1}{2.4}\,\frac{1}{2^4} + \frac{1}{3.5}\,\frac{1}{2^5} - \&c. \\ \cos\theta - \frac{1}{2^2}\cos 2\theta + \frac{1}{3^2}\cos 3\theta - \&c.\end{array}\right\}, \textit{ ad infinitum.}$$

5. Given the radius of the circumscribed circle and the three angles of a triangle; find expressions for the three sides.

6. Develope $\sin(\alpha + \beta x + \gamma x^2)$ in a series of the form $A + Bx + Cx^2 + Dx^3 + \&c.$

7. In an ellipse in which the semi-axes are CA, CB, and the abscissa and ordinate CM and MP, in MP take

$$MQ = \frac{CB^2}{CM + MP},$$

trace the curve which is the locus of Q; find its maximum and minimum ordinates, and the angles made by its two extremities with the axes.

8. Integrate the differentials;

$$\frac{x^4\,dx}{\sqrt{2ax - x^2}}, \quad \frac{\sin^2 x\,dx}{\cos^4 x}, \quad e^{az}\cos^m z\,dz;$$

and the differential equations;

$$\sqrt{x}\,d x+\sqrt{y}\,d y=\sqrt[4]{x y}\,d y,\quad \frac{d^2 y}{d x^2}+\frac{y}{x^2}=x^3+a x^2.$$

9. Give a construction depending upon the cycloid, for determining an arc equal to its cosine.

10. State the most recent and approved experiments, whereby it is ascertained that the decrement of velocity arising from friction is the same for all velocities.

11. A bent lever, of which the arms are a and b, and the angle θ, makes small oscillations in its own plane; the length of the isochronous simple pendulum is

$$\frac{2}{3}\,\frac{a^3+b^3}{\sqrt{a^4+b^4+2\,a^2 b^2\cos\theta}}.$$

12. Find the actual velocity of the point P, (Newt. Book I. Sect. vii. Prop. 39.) the force tending to C being supposed to vary directly as the distance.

13. The force in an orbit $\propto \dfrac{m}{\sqrt{a^2+r^2}}$, where r is the radius vector; find the angle between the apsides when the orbit is nearly circular.

14. Why do objects appear further off and smaller, when viewed through the wrong end of a telescope?

15. A given hemispherical vessel filled with fluid is whirled round its vertical axis, so that the surface of the fluid which remains, touches the lowest point of the hemisphere; find the angular velocity and the quantity of fluid remaining.

16. Compare the portion of the surface of the Earth illuminated by the Sun in perigee with that illuminated in apogee, taking into account the magnitude of the Sun.

17. A given quantity of matter is to be formed into a cone; find its form, that its attraction on a particle at its vertex may be a maximum, the attraction of each particle varying as $\dfrac{1}{D^2}$.

18. If the true centre of the Moon's orbit move uniformly in a circle about the mean centre, the result is a change of the Moon's place of the form $m \sin 2\left\{(☽ - ☉) - A\right\}$, where A is the Moon's anomaly.

MONDAY AFTERNOON.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

First and Second Classes.

1. Explain the method of drawing a tangent plane at any proposed point of a given curve-surface, and find the equation to that plane in the case of an ellipsoid.

2. Explain the relation which exists between the curves which the complete integral of any differential equation of the first order represents, and that which is defined by a particular solution of that equation; and shew how the particular solution may be deduced from the complete integral. Exemplify in the case where $y = x \tan \alpha - \dfrac{x^2}{4\,h \cos^2 \alpha}$ is the complete integral, α being the arbitrary constant.

3. Prove the following theorem in finite differences;

$$\Sigma\, u_x v_x = u_x \Sigma\, v_x - \Delta\, u_x \Sigma^2 v_{x+1} + \Delta^2 u_x \Sigma^3 v_{x+2} - \&c.$$

and apply it to find the sum of x terms of the series

$$1 \,.\, 2 \,.\, 3\, a - 2 \,.\, 3 \,.\, 4\, a^2 + 3 \,.\, 4 \,.\, 5\, a^3 - \&c.$$

4. State and explain *D'Alembert's* Principle, and apply it to determine the pressure on the axis about which a body revolves when acted on by a single force in a plane perpendicular to the axis.

5. Give *Newton's* construction for determining the path of a projectile acted upon by gravity in a medium whose resistance $\propto$ velocity, and apply the differential equations of motion to determine the actual equation.

6. Shew from *Newton's* construction for determining the horary increment of the area described by the Moon in a circular orbit round the earth at rest, that the velocity generated by the tangential ablatitious force between quadrature and syzygy is to that which would be generated in the same time by the mean addititious :: 3 : π.

7. Investigate the general equation of equilibrium of any fluid; and shew from the equation that the resultant of the forces at any point in the surface of a fluid incompressible and perfectly free, is a normal to the surface.

8. If the whole force at the pole of an oblate spheroid be to that at the equator as the equatorial radius to the polar, and to any point within the spheroid canals of any form be drawn, the pressure on that point will be the same whatever be the form or direction of the canal.

9. Apply *Lagrange's* Theorem to the determination of the first three terms of the series expressing the true anomaly in terms of the mean, the series ascending by powers of ϵ and the anomaly being measured from the perihelion.

MONDAY AFTERNOON.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

Third and Fourth Classes.

1. Shew that the limiting equation has at least as many possible roots as the original equation, wanting one; and determine the nature of the roots of the equation

$$x^7 - a^5 x^2 + c^7 = 0.$$

2. State *Napier's* rules for the solution of right-angled spherical triangles, and prove the two cases in which the complement of the hypothenuse is the middle part.

3. Shew generally how to find the evolute of any curve whose equation is given, and find that of the common parabola.

4. Solve the differential equation

$$(a + bx + cy)\,dx = (a' + b'x + c'y)\,dy.$$

5. When any number of bodies move uniformly in straight lines in different planes, their centre of gravity also moves uniformly and in a straight line.

6. Having given the moment of inertia round any axis passing through the centre of gravity of a body, to determine that round any axis parallel to the former.

7. To determine the horary motion of the Moon's nodes in a circular orbit. (Newton, Book III. Prop. 30.)

8. If the object placed before a spherical reflector be a straight line, the image will be a conic section. Prove this, and shew how the different parts of the image are formed, when the object is placed between the principal focus and the surface of the reflector.

9. A globe is projected vertically upwards with a given velocity c, in a medium where the resistance is $= k \times (\text{vel.})^2$, and is acted on by gravity; determine the relation between the time, space and velocity.

10. Given three distances of a planet from the Sun, and the corresponding arguments of latitude, to find the place of the perihelion, and the true anomaly at the first observation.

TUESDAY MORNING.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

First Class.

1. Given a solution of a differential equation of the first order, find whether it is included in the complete integral.

2. Given the solution of the equation $\frac{d^2 u}{d v^2} + u = 0$, solve

$$\frac{d^2 u}{d v^2} + u + a \cos v = 0,$$

by the method of the variation of parameters.

3. Determine $\Delta^n u_x$ in a series involving

$$u_{x+n},\ u_{x+n-1},\ u_{x+n-2},\ \&c.$$

4. When any number of forces act on a body, shew that the plane on which the sum of the projections of the *moments* is a maximum, is perpendicular to the planes with respect to which this sum is 0.

5. Find the attraction of a spheroid of finite eccentricity on a particle in its equator.

6. Explain fully the principles on which *Newton* calculates the correction in the motion of the nodes due to the unequable description of areas, and shew that the mean decrement when the nodes are in quadratures is equal $\frac{1}{4}$ decrement in syzygy.

7. A body may revolve in the equiangular spiral in a medium of which the density is inversely as the distance from the centre by a force varying inversely as the square of the distance from the centre. (Newton, Book II. Prop. 15). Prove this geometrically and analytically.

8. If a uniform force act upon a body tending to give it a motion of rotation about an axis always perpendicular to the axis about which it is at each instant revolving, and always in the

same plane, the angular velocity is unaltered. Shew hence that the angular velocity of the Earth is not affected by the action of the Sun and Moon.

9. Construct for the place of high water in a given position of the Sun and Moon, and find an expression for the actual height of the compound tide.

10. Mention the facts from which it appears that the phenomena of the extraordinary ray in a double refracting crystal can be accounted for on the supposition of a repulsive force emanating from the axis.

TUESDAY MORNING.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

Second and Third Classes.

1. Approximate to the greatest root of the equation

$$x^3 - 7x = 1.$$

2. Having given the equation to an ellipse referred to its principal axes, transform it into one in which the axes are inclined at an angle θ, and in which the axis of y' makes with that of y, a given angle ϕ. Find also the relation between ϕ and θ when the transformed equation is of the same form with the original equation, and shew that in this case each of the new axes is parallel to the tangent drawn at the extremity of the other.

3. Shew that in a curve surface the sections of the greatest and least curvature are at right angles to each other.

4. Shew that every recurring series may be resolved into a certain number of geometric series, and exemplify the method by resolving the following series, and finding the sum of any number of its terms

$$1 + 4 + 18 + 80 + 356 + \ldots\ldots$$

5. If gravity act upon a system of bodies $m', m'', \ldots\ldots$ and $h', h'', \ldots\ldots$ be the vertical spaces described, and $v', v'', \ldots\ldots$ be the actual velocities of the bodies, prove that

$$m'v'^2 + m''v''^2 + \ldots\ldots = 2g(m'h' + m''h'' + \ldots\ldots).$$

6. Required the geometrical focus of a thin pencil of rays after being refracted at a curved surface.

7. Find the place of a body in a parabolic orbit at any assigned time. (Newton, Book I. Prop. 30.)

8. Find the horary variation of the inclination of the lunar orbit to the plane of the ecliptic. (Newton, Book III. Prop. 34.)

9. Shew how to determine the altitudes of mountains by the barometer, and explain the corrections to be applied in consequence of a change in temperature.

10. The centre of gravity of the Earth and Moon describes an orbit round the Sun much more nearly elliptical than that described by the Earth or Moon.

TUESDAY MORNING.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

Fourth Class.

1. Prove the rule for transforming a number from one scale of notation to another. Transform 1828 to local value 3.

2. Transform the equation $x^3 - 6x^2 + 11x - 6 = 0$, whose roots are α, β, γ, into the equation whose roots are

$$\frac{1}{\alpha^2+\beta^2}, \quad \frac{1}{\alpha^2+\gamma^2}, \quad \frac{1}{\beta^2+\gamma^2}.$$

3. In the expression $y = 2x^3 - 15x^2 + 36x$, find for what values of x, y is a maximum or minimum, and in each case which.

4. Integrate $\dfrac{dx}{\sqrt{1+x+x^2}}$, and $\dfrac{x^{2n}\,dx}{1+x^2}$.

5. A pendulum is taken to the top of a hill; how many seconds a day does it lose?

6. Enunciate and prove *Newton*, Lemma 9.

7. If diverging rays fall upon a concave spherical surface of a rarer medium, to find the geometrical focus of refracted rays.

8. Construct the common pump, and find the height which the water rises at each stroke.

9. Construct a vertical south dial for a given latitude.

10. Shew how a planet, superior or inferior, may have its motion direct or retrograde, or may be stationary.

TUESDAY AFTERNOON.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

First Class.

1. Shew the method of integrating $Pp + Qq = R$, when neither $P\,dy - Q\,dx = 0$, nor $P\,dz - R\,dx = 0$, are integrable separately and independently, and explain the process fully.

2. Define conical surfaces, and investigate their general equation.

3. Find the variation of $\int V\,dx$, and explain the use of that part of the result which is without the integral sign, and exemplify it by finding the shortest distance between two given straight lines not in the same plane.

4. In a lottery of m tickets, n of which are prizes, if p tickets be drawn at each time, what is the probability that all the prizes will be drawn after q drawings?

5. Give an analysis of the reasoning by which *Newton* explains the theory of the tides, and deduce a numerical comparison between the force of the Sun on the tides, and the force of gravity.

6. If a body float on a fluid, determine its stability at a small angle of inclination from a given position of equilibrium; and the time of one of its small oscillations.

7. Find the general equation for the motion of a vibrating cord.

8. Explain the method of determining the Sun's parallax by observations made on the transit of Venus over the Sun's disc.

TUESDAY AFTERNOON.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

Second and Third Classes.

1. From the equation $z = f\left(\frac{y^2 - x^2}{x}\right)$ to eliminate by differentiation the quantity $f\left(\frac{y^2 - x^2}{x}\right)$.

2. If the equations to two planes be $\left\{\begin{matrix} a x + b y + c z = d \\ \alpha x + \beta y + \gamma z = \delta \end{matrix}\right\}$, find the angle made by the planes.

3. $y^2 = \frac{x^3 - a^3}{x + a}$; trace the curve and draw its asymptotes.

4. If u be a homogeneous function of x and y of m dimensions, shew that $m u = \frac{d u}{d x} x + \frac{d u}{d y} y$; and hence find a factor which renders $M d x + N d y = 0$ integrable, M and N being homogeneous functions.

5. At points of greatest and least curvature, the osculating circle will have with the curve a contact of a higher than the second order.

6. Find the velocity acquired by a cylinder unrolling and descending vertically through a given space.

7. Compare the times of bodies oscillating in a hypocycloid, revolving about the centre, and falling to the same centre, the force varying as the distance. (Newton, Book I. Prop. 52. Cor. 3.)

8. Find the attraction of a spherical shell in which the attraction of each particle $\propto \dfrac{1}{(\text{dist})^2}$ according to *Newton's* method, and analytically.

9. Find the aberration of a given star in R. A. in terms of the R. A., declination, obliquity and Sun's longitude.

10. If m be the maximum lunar nutation in N. P. D., l the longitude of the Moon's ascending node, shew that when the longitude $= l'$, the nutation $= m \cos (l' - l)$.

TUESDAY AFTERNOON.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

Fourth Class.

1. Find the cosine of the angle contained between two straight lines whose equations are

$$y = a\,x + b, \text{ and } y = a'\,x + b'.$$

2. Shew how to determine the value of a vanishing fraction in all cases; and find the value of

$$\frac{1 - \dfrac{2\,x}{\pi}}{\text{cotan}\, x}, \text{ when } x = \frac{\pi}{2},$$

$$\text{and of } \frac{\sqrt{a} - \sqrt{x} + \sqrt{a - x}}{a - \sqrt{2\,a\,x - x^2}}, \text{ when } x = a.$$

3. Integrate the following differentials:

$$\frac{d\,x}{(x + a)\,(x^2 + a^2)}, \quad \frac{x^5\,d\,x}{\sqrt{1 - x^2}}.$$

4. Sum the following series:

$$\left.\begin{array}{l} \dfrac{1}{1\,.\,2}+\dfrac{3}{2\,.\,3}+\dfrac{5}{3\,.\,4}+\dfrac{7}{4\,.\,5}+\ldots. \\ 1^3+2^3+3^3+4^3+\ldots\ldots\ldots\ldots \end{array}\right\}\text{ to } n \text{ terms.}$$

5. Divide a cylinder filled with fluid into two such parts, that the times of emptying the fluid contained in each, through a small orifice in the base, may be the same.

6. A body descends in a straight line towards a centre of force varying as the distance, find the velocity acquired in descending through a given space, and the time of descent. (Newton, Book I. Prop. 38.)

7. Find the difference of the forces in the fixed and moveable orbits. (Newton, Book I. Prop. 44.)

8. Prove analytically that the areas described by a body about any centre of force are in the same plane, and proportional to the time.

9. Find the precession in north polar distance.

10. If a small pencil of parallel homogeneal rays be refracted into a sphere, and the ratio of the sine of incidence to the sine of refraction be known, to find at what angle the rays must be incident, that they may emerge parallel after any given number of reflections within the sphere.

1829.

MODERATORS,

MR. JEFFREYS, St. John's. MR. BOWSTEAD, Corpus.

EXAMINERS,

MR. WHEWELL, Trinity. MR. KING, Queen's.

FRIDAY MORNING.

QUESTIONS IN PURE MATHEMATICS.

First, Second, Third and Fourth Classes.

1. IN a circle the angle in a semi-circle is a right angle: but the angle in a segment greater than a semi-circle is less than a right angle, and the angle in a segment less than a semi-circle is greater than a right angle.

2. Extract the square root of 235.6 to two places of decimals: and the cube root of .000079507.

3. Having given two sides and the included angle of a plane triangle, find the angles at the base: find also the third side by an independent method, and adapt the trigonometrical expressions to logarithmic computation.

4. Prove the formula

$$\sin(a+b) = 2\sin a - \sin(a-b) - 4\sin a \sin^2\frac{b}{2};$$

and explain fully its use in the construction of the Trigonometrical Canon.

5. In an ellipse prove that

$$CP^2 + CD^2 = AC^2 + BC^2.$$

6. Prove that the chord of curvature of an hyperbola through the focus

$$= \frac{2\,CD^2}{AC}.$$

7. Expand a^x in a series ascending by the powers of x.

8. Prove that the *Napierian* logarithm of $N + z$

$$= \text{Nap. log}\, N + 2\left\{\frac{z}{2\,N + z} + \frac{1}{3}\,\frac{z^3}{(2\,N + z)^3} + \&\text{c.}\right\};$$

and from this formula shew how the logarithm of a number of six places of figures may be found from a table computed only to five places of figures.

9. Prove the rule for finding the greatest common measure of two algebraical quantities, and apply it to find the greatest common measure of

$$x^3 - 8\,x^2 - 12\,x + 144 \text{ and } 3\,x^2 - 16\,x - 12.$$

10. Prove the Binomial Theorem for any value of the index.

11. A ratio of greater inequality is diminished and of less inequality increased, by adding any quantity to both its terms.

12. The reciprocals of quantities in harmonical progression are in arithmetical progression.

13. Every equation, whose roots are possible, has as many changes of sign from + to − and from − to +, as it has positive roots; and as many continuations of the same sign from + to + and from − to −, as it has negative roots.

14. If the equation $x^3 - p\,x^2 + q\,x - r = 0$ has two equal roots, one of them is $\frac{9\,r - p\,q}{6\,q - 2\,p^2}$; but the converse is not necessarily the case.

15. Explain the method of finding those roots of an equation which are whole numbers, by the *Method of Divisors*, and apply it to solve the equation

$$x^3 - 9\,x^2 + 22\,x - 24 = 0.$$

16. State *Napier's* rules for the solution of right-angled spherical triangles, and prove them when the complement of the hypothenuse is the middle part.

17. Find the equation to the curve in which the distance of any point from a given fixed point. is equal to the perpendicular drawn from the same point in the curve upon a given line.

18. Find the present value of an annuity to be paid for n years, allowing compound interest.

19. Having given the equation to a straight line, find the equation to another straight line drawn perpendicular to it from a given point; find also the length of the perpendicular.

20. Every square number is of the form $5n$ or $5n \pm 1$.

FRIDAY AFTERNOON.

QUESTIONS IN NATURAL PHILOSOPHY.

First, Second, Third and Fourth Classes.

1. If two weights acting perpendicularly upon a straight lever on opposite sides of the fulcrum, or two forces in opposite directions on the same side of it, are inversely as their distances from the fulcrum, they will balance each other.

2. If on an isosceles wedge of which the angle is 2α, a power P acting perpendicular to the base, balance a resistance W acting on each of the sides in a direction making an angle ι with a perpendicular to the side,

$$P : W :: \sin \alpha : \cos \iota.$$

3. When a system is in equilibrium, if a small motion be given to its parts, the centre of gravity will neither ascend nor descend.

4. If two imperfectly elastic bodies impinge obliquely on each other with given velocities and directions, find the velocities and directions of their motions after impact.

5. If a body acted on by gravity be projected from a given point A with a given velocity so as to strike a given point Q, find the direction of projection; and if AI bisects the angle which AQ makes with the vertical, shew that there are two such directions equally inclined to AI.

6. Explain how fluids press equally in all directions; and from this shew that in all tubes communicating with each other, a fluid will stand at the same altitude.

7. Shew that when a body floats in a fluid, the weight of the body is equal to that of the fluid displaced, and that their centres of gravity are in the same vertical line; and hence explain the construction and use of the hydrometer.

8. If a given quantity of air be left in the tube of a barometer, find the depression below the standard altitude.

9. If Q and q be the conjugate foci of rays incident nearly perpendicular on a spherical reflector; E the centre and F the principal focus,

$$FE^2 = FQ \, . \, Fq.$$

10. A *convex* spherical refracting surface of a *denser*, and a *concave* of a *rarer* medium, diminish the divergency and increase the convergency of all pencils of rays incident nearly perpendicularly, unless the focus of incident rays be between the surface and centre of the refractor.

11. Construct *Gregory's* telescope. Draw accurately the course of the extreme ray. Express the magnifying power in terms of the focal lengths and the distance between the principal foci of the reflectors.

12. Prove and explain *Newton*, Lemma II.

13. Enunciate and prove *Newton*, Lemma X; and if the force be constant shew that it is true when the time is finite.

14. Find the law of force acting in parallel lines by which a body will be made to describe a portion of the circle. (Newton, Book I. Prop. 8.)

15. Find the law of force tending to the focus, by which a body may be made to describe an ellipse. (Newton, Book I. Prop. 11.)

16. Explain the nature of centrifugal force, and shew that in a body revolving about a centre it varies inversely as the cube of the distance.

17. If a body be projected from a given point with a given velocity in a given direction, and acted on by a force which varies inversely as the square of the distance; find the conic section described.

18. Explain the causes of change of seasons, and of the different lengths of day and night. Shew that the full Moon in winter is longer above the horizon than in summer.

19. Explain by what observations the path of the Sun among the fixed stars is determined.

20. Shew that the aberration of a star takes place towards a point of the ecliptic 90° before the Earth's place, and that it varies as the sine of the angle of the Earth's way.

21. Explain the cause of twilight, and shew how its duration may be found from the declination of the Sun, and the latitude of the place. Find also on what day it is shortest at a given place.

EVENING PROBLEMS.—Mr. Jeffreys.

1. Find the value of the circulating decimal 3.42753753 &c. also, perform the same operation with the fraction 45.2534534 &c. where the radix is 6.

2. The sides of a triangle are in arithmetical progression, and its area is to that of an equilateral triangle of the same perimeter as 3 : 5. Find the ratio of the sides, and the value of the largest angle.

3. A cone and sphere of given weights, support each other between two given inclined planes, the cone resting on its base. Determine what must be the vertical angle of the cone, that the equilibrium may subsist.

4. Trace the curve, of which the equation is $y^5 = a x^4 + m x^5$; draw its asymptote, and determine its singular points.

5. An hemisphere with its base downwards is filled with fluid, and inclined at a given angle to the horizon: the base being moveable about a tangent at its upper extremity, find the force which applied at the centre will keep it at rest.

6. If tangents be drawn from a given point to each of a given system of curves, shew generally how to determine the curve which is the locus of all the points of contact; and apply the method when tangents are drawn from a given point, to a system of concentric similar ellipses.

7. Find an expression in terms of the sides of a spherical triangle for the arc drawn from one angle C bisecting the opposite side c, and adapt the expression to logarithmic computation.

8. If the object placed before a lens, or a spherical reflecting or refracting surface, be a conic section, of which the focus and axis coincide respectively with the centre and axis of the reflector or refractor; the image will be a conic section. Prove it in each case, and find the eccentricity and axes of the image.

9. Find the equation between the angle and radius vector in a spiral, in which the radius vector is always equal to n times the chord of curvature drawn through the pole. Find also the value of the radius of curvature in such a spiral.

10. The style of a horizontal dial, which is accurately graduated for a given place, is bent through a small given angle δ; if a be the hour angle at which there is no error in the time denoted by the dial, find the error in the time denoted

at any hour angle h, on a given day; and shew that at six o'clock the error is independent of the latitude of the place, and vanishes at sun-set.

11. To inscribe the greatest ellipse in a given semi-circle, one axis of the ellipse being parallel to the diameter of the semi-circle.

12. If there be n bags containing each a white and b black balls, and a ball be drawn out of each successively, shew how to determine beforehand what is the most probable number of white balls that will be drawn; and apply the process to the case of 12 bags containing each 2 black and 7 white balls.

13. Explain the use of the Astronomical Clock, and shew how we may determine whether it goes uniformly at all hours. Also, explain clearly why the *mean daily rate* determined by the assistance of tables, differs from that determined by direct observations on the transits of stars, and state for what stars this disagreement is perceptible.

14. A body urged towards a plane by a force varying as the perpendicular distance from it, is projected at right angles to the plane from a given point in it with a given velocity. Find what force must act at the same time on the body parallel to the plane, that it may move in a given parabola having its axis in the plane; and determine the circumstances of the motion.

15. If the distance CP in an ellipse be a mean proportional between the semi-axes, prove that the semi-conjugate CD divides the quadrant of the ellipse into two arcs, whose difference is equal to the difference of the semi-axes.

16. AM, MN, NP being the co-ordinates to the point P in a curve surface, find the equation to the surface when the foot of the normal lies always in the centre of gravity of the triangle AMN.

17. Integrate the following differentials and differential equations:

$$\frac{dx}{\sqrt[3]{1+x}+\sqrt[5]{1+x}}, \quad \frac{dx}{(x^2+1)\sqrt{x^2-2}},$$

$x^6 \sin x \, dx$ from $x = 0$ to $x = \frac{\pi}{2}$, $\frac{dx}{dy} = xy + x^2 y^3$,

$$2x\frac{dz}{dx} + y\frac{dz}{dy} = 2xy,$$

and $\frac{d^4 y}{dx^4} - \frac{d^2 y}{dx^2} + y = 0$, in a rational form;

and sum the series

$$\frac{16}{2.3.4} - \frac{2.21}{3.4.5.3} + \frac{2^2.26}{4.5.6.3^2} - \&c.$$

to n terms and to infinity.

18. Find the whole longitudinal aberration when a pencil of parallel rays passes through a plano-convex lens of inconsiderable thickness, the rays being incident on the convex side in a direction parallel to the axis of the lens.

19. If a solid hemisphere rests on its base wholly immersed in a fluid of less specific gravity than itself, whilst the fluid flows horizontally against it with a given velocity, find the whole pressure of the hemisphere against the bottom of the vessel, the solid being kept at rest by the friction.

20. Find the attraction on a particle placed within an heterogeneous spheroidal shell of small eccentricity, the density being the same throughout concentric spheroidal surfaces of different eccentricities, and the internal and external surfaces being of given eccentricity, and the density uniform throughout them.

21. A uniform rod is oscillating about one extremity; find the tendency of the *vis inertiæ* in any given position to bend the rod at any point, and determine at what point that tendency is the greatest.

22. A body is oscillating in a cycloid, in a medium where the resistance varies as the $(\text{vel.})^2$, and the density varies inversely as the arc measured from the lowest point: prove by a method similar to *Newton's* in Book II. Prop. 26. that the time of descent to the lowest point will be the same from all altitudes; and apply the integral calculus to find the whole time, supposing the resistance at the highest point corresponding to any velocity v, to be less than $\frac{v^2}{l}$, where l is the length of the pendulum.

23. If a straight line DCP be made to revolve about C, and cut the curve PP_1P_2 in as many points as it has dimensions; and if $\frac{1}{CD^2}$ be made always equal to $\frac{1}{CP^2}+\frac{1}{CP_1^2}+$ &c., the locus of the point D will be a conic section whose centre is C.

24. Prove that the motion of a body P round T, when disturbed by a body S, is determined by the equation

$$\frac{d^2x}{dt^2}+\frac{P+T}{r^3}x+\frac{dR}{dx}=0,$$

together with two similar equations in y and z, where $r=PT$,

$$\text{and } R=\frac{S\,.\,r}{ST^2}\cos\angle STP-\frac{S}{SP};$$

and by means of them exemplify fully the method of determining the variation of the elements of P's orbit arising from the disturbance, when that disturbance is small.

SATURDAY MORNING.

QUESTIONS IN PURE MATHEMATICS.

First, Second, Third and Fourth Classes.

1. If two triangles have the sides about equal angles reciprocally proportional, they are equal.

2. If a straight line be at right angles to two straight lines at their point of intersection, it is at right angles to the plane passing through them.

3. Investigate a true rule for the equated time of payment of two sums due at different times. Who is the gainer by the common rule?

4. Expand the n^{th} power of sin A in terms of sines and cosines of multiples of A; and write down the last term with its proper sign when n is of the form $4m + 2$.

5. State the construction of the polar triangle, and shew that its sides and angles are respectively the supplements of the angles and sides of the original triangle.

6. If a line, intersecting an hyperbola in the point P and its asymptotes in R, r, move parallel to itself, the rectangle $RP \,.\, Pr$ is constant.

7. The section of a right cone made by a plane will be an ellipse, hyperbola or parabola. Prove this, and determine the position of the plane for each case.

8. Shew that any recurring equation of $2m$ or $2m + 1$ dimensions may be solved by an equation of m dimensions.

9. In the solution of a biquadratic by *Des Cartes's* method, whatever root of the reducing cubic is employed, the same values of the roots of the biquadratic will be obtained.

10. If $\dfrac{N}{N'}$ and $\dfrac{P}{P'}$ be successive approximations to the value of a continued fraction,

$$NP' - PN' = \pm 1.$$

11. Investigate the formulæ for the transformation of rectangular co-ordinates to oblique co-ordinates in the same plane; and hence, having given the equation to a parabola referred to its principal axis, find its equation referred to any other diameter; the abscissa being measured from a point in the curve, and the ordinate being parallel to the tangent at that point.

12. Find the limiting ratio of the corresponding *increments* of $\sqrt{a^2 + x^2}$ and x; and those of $\dfrac{e^x + 1}{e^x - 1}$ and x.

13. Expand $\sin x$ and $\cos x$ by *Taylor's* Theorem; and find limits of the value of the terms after the n^{th} term in each case.

14. Find the rectangular equation to the conchoid of *Nicomedes*, and draw a tangent to the curve.

15. Trace the curve of which the equation is

$$y = x \cdot \frac{x - a}{x - 2a};$$

and find its maximum and minimum ordinates.

16. Define a differential; and hence find the differential of a solid of revolution. Also, apply this to prove that the sphere is $\frac{2}{3}$ of the circumscribing cylinder.

17. Find the sum of the series

$$1 \cdot 3 \cdot 4 + 2 \cdot 4 \cdot 5 + 3 \cdot 5 \cdot 6 + \text{\&c. to } n \text{ terms},$$

$$\text{and of } \frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 6} + \text{\&c. } \textit{in infinitum}.$$

18. Integrate $\dfrac{dx}{x^3 + 1}$; and shew that between the limits $x = 0$ and $x = 1$, it is

$$\frac{1}{3} \log 2 + \frac{\pi}{3\sqrt{3}}.$$

SATURDAY AFTERNOON.

QUESTIONS IN NATURAL PHILOSOPHY.

First, Second, Third and Fourth Classes.

1. Find the ratio of the power to the weight in that system where each pulley hangs by a separate string; first, when the strings are parallel; secondly, when they are not.

2. Find the resultant of any number of parallel forces acting on a rigid body, and shew that they cannot in all cases be reduced to a single force which shall have the same effect.

3. Determine the equation to the catenary, the force of gravity being supposed constant.

4. Find the limit of the velocity communicated by a body A to C, through an indefinite number of mean proportionals between A and C, the bodies being supposed perfectly elastic.

5. Explain *D'Alembert's* Principle, and apply it to determine the motion of two bodies connected together by a wheel and axle; the inertia of the machine being taken into account.

6. At similar points in similar curves described round centres of force similarly situated, the forces are as the squares of the velocities directly, and the distances inversely.

7. Find the place of a body in an elliptic trajectory after a given time. (Newton, Book I. Prop. 31.)

8. State and prove the proportion which *Newton* has given in Book I. Prop. 44, Cor. 1, for determining the actual value of the difference of the forces in the fixed and moveable orbits, and apply it to determine the whole force on p, when the fixed orbit is an ellipse, and the force in the focus.

9. If P and S attract each other, the curve which P describes relatively to S, may be described by P round S fixed. (Newton, Book I. Prop. 58.)

10. Prove that the pressure upon any portion of a vessel filled with a fluid of uniform density is equal to the weight of a column of fluid whose base is the area of the surface pressed, and altitude the perpendicular depth of its centre of gravity below the surface of the fluid; and find the whole pressure on the surface of a spherical segment filled with fluid.

11. If a floating body revolve round a horizontal axis and so pass through all its positions of equilibrium; they will be alternately stable and unstable.

12. If the distances above the surface of the Earth increase in arithmetical progression, the corresponding densities of the air will decrease in geometrical progression; the force of gravity being invariable.

13. Explain the formation of the primary and secondary rainbows, and why the red arc is exterior in the former case and interior in the latter.

14. Converging rays are incident upon a concave spherical reflector; find the longitudinal and lateral aberrations.

15. Explain *Flamstead's* method of determining the right ascension of the Sun, by observations made near the equinoxes; and show that it will serve to determine the place of the equinox.

16. On a given day and hour, find the azimuth of the ascending point of the ecliptic, and the longitude and altitude of the nonagesimal degree.

17. Find the times of the beginning and end of a lunar eclipse, and the number of digits eclipsed.

18. Prove that the stereographic projection of any circle on the sphere, is a circle; and find the centre and radius of the circle.

EVENING PROBLEMS.—Mr. Bowstead.

1. A Banker borrows money at $3\frac{1}{2}$ per cent. per annum, and pays the interest at the end of the year: he lends it out at the rate of 5 per cent. per annum, but receives the interest quarterly, and by this means gains 200*l.* a year. How much does he borrow?

2. Given $\tan 3A = n \tan A$. Find A in terms of n: find also the value of n that A may be 15°.

3. AP is the arc of a conic section, of which the vertex is A: PG the normal and PK a perpendicular to the chord AP, meet the axis in G and K. Shew that GK is equal to half the latus rectum.

4. All objects from the zenith to the horizon are visible to an eye under water, and appear to be bounded by a conical surface of which the eye is the vertex. Explain this, and, having given the refracting power of water, find the vertical angle of the conical surface. State also what is seen beyond the limits of the conical surface.

5. The equation to a conic section is

$$5y^2 + 2xy + 5x^2 - 12x - 12y = 0.$$

Find its centre, and the magnitudes and positions of its principal axes.

6. A weight W is suspended from a point P of an uniform catenary APA'. O and O' are the lowest points of two uniform catenaries, of which AP and $A'P$ are parts. Shew that W is equal to the *difference* or *sum* of the weights of the portions OP, $O'P$ of the catenaries, according as AP and $A'P$ are *one*, or *both* less than a semi-catenary.

7. The co-efficient of x^n in the expansion of $(1 + x + 2x^2 + 3x^3 + \ldots\ ad\ inf.)^2$ is equal to $\dfrac{n^3 + 11n}{6}$.

8. A body revolving in a given ellipse, force in focus, leaves the higher apse; and when it arrives at the lower apse A, the absolute force is suddenly altered, so that the body describes a similar ellipse, of which A is the higher apse; and a like alteration takes place when the body arrives at the lower apse of the new ellipse, and so on at each successive apse. Find the time to the centre of force.

9. The heights of the ridge and eaves of a house are H and h, and the roof is inclined at 30° to the horizon. Find where a sphere *rolling* down the roof from the ridge will strike the ground, and also the time of descent from the eaves.

10. If $A_1, A_2 \ldots\ldots. A_n$ and $B_1, B_2 \ldots\ldots. B_n$ be two series of positive numbers arranged in order of magnitude, of which A_1 and B_1 are respectively the greatest, shew that

$$\frac{A_1}{B_1} + \frac{A_2}{B_2} + \ldots. + \frac{A_n}{B_n} \text{ is less, and } \frac{A_1}{B_n} + \frac{A_2}{B_{n-1}} + \ldots. + \frac{A_n}{B_1}$$

greater, than if the denominators $B_1, B_2 \ldots.. B_n$ be arranged in any other order under $A_1, A_2 \ldots.. A_n$.

11. A hemispherical vessel filled with fluid revolves round its vertical diameter with such an angular velocity that half the fluid is thrown out: required the pressure on the surface.

12. The first of February of the present year (1829) falls on a Sunday. Find generally when this will happen again; and write down all the years in which it will occur during the present century.

13. The content of any segment of a right or oblique prismatic solid, is equal to the area of one end of the segment, multiplied into the perpendicular let fall upon it from the centre of gravity of the area of the other end.

14. A river, of which the breadth is a, flows with a velocity u, and a swimmer, whose velocity is $n u$, always aims at a mark on the farther bank directly opposite to the place where he entered

the river. Find the curve in which he swims, and shew that the time of his arriving at the mark is equal to

$$\frac{n a}{(n^2 - 1) u}.$$

15. Two vessels, of which the capacities are a and b, are filled, the one with wine and the other with water; equal quantities c are taken from each and poured into the other, and this operation is repeated n times. Find the quantities of wine and water remaining in each vessel.

16. Prove that the integral of $\frac{x^{m-1}\,dx}{x^n + 1}$ between the limits $x = 0$ and $x = \infty$, is equal to $\frac{\pi}{n \sin \frac{m}{n} \pi}$, n being greater than m.

17. A uniform rod, of which the elasticity is e, falls upon a smooth horizontal plane: given the altitude from which it falls, and its inclination to the horizon, find its motion after rebounding.

18. If rays are incident upon a common looking glass in an oblique direction from a candle, one faint image is observed before the principal image, and a row of them behind it. Explain this; and find the caustic formed by the rays emergent from the glass after reflection at the quicksilver, the thickness of the glass and its *refracting* power being given.

19. A and B sit down to cards, the former having p shillings and the latter q, and they agree each to stake a shilling on every game, and to play till one has lost all his money. Find the value of the expectation of each before they begin to play, supposing the skill of A : that of B :: m : n.

20. Integrate

$$\frac{dx}{(a^2 + x^2)^4}, \quad \frac{dx}{a + b \sin x + c \cos x}, \quad \frac{dy}{dx} = \frac{10 + 6y - 4x}{6x - 9y + 3},$$

$$\left.\begin{aligned}\frac{d^2 y}{d t^2} + 2\frac{d^2 x}{d t^2} &= 2y - 3x - 1\\ \frac{d^2 x}{d t^2} - 2\frac{d^2 y}{d t^2} &= x + 16y - 3\end{aligned}\right\},$$

and sum the series

$$\frac{1}{2^2 - 3^2} + \frac{1}{4^2 - 3^2} + \frac{1}{6^2 - 3^2} + \ldots . \text{ to } n \text{ terms, and to infinity;}$$

$$1 . 3 \sin \theta + 3 . 5 \sin 3\theta + 5 . 7 \sin 5\theta + \ldots . \text{ to } n \text{ terms;}$$

$$\frac{1}{1^3 . 3^3} + \frac{1}{1^3 . 5^3} + \&\text{c.} + \frac{1}{3^3 . 5^3} + \&\text{c. } \textit{ad infinitum.}$$

21. If x_1, y_1, z_1 be the distances from the origin of the co-ordinates at which a tangent plane to a curve surface cuts the axes x, y, z, and if $x_1{}^n + y_1{}^n + z_1{}^n = a^n$, prove that the equation to the surface touched is

$$x^{\frac{n}{n+1}} + y^{\frac{n}{n+1}} + z^{\frac{n}{n+1}} = a^{\frac{n}{n+1}}.$$

22. $u_s \, u_{s+\pi} = a\,(u_s + u_{s+\pi})$. Give a complete solution of this equation, and determine the particular values of the constants when it is the equation to a conic section about the focus.

23. If u_1, u_2 be values of u which satisfy the differential equation

$$\frac{d^2 u}{d t^2} + M\frac{d u}{d t} + N u = 0,$$

M and N functions of t, shew that the integral of

$$\frac{d^2 u}{d t^2} + M\frac{d u}{d t} + N u + \Pi = 0 \text{ is } c_1 u_1 + c_2 u_2 - u_1\int\frac{u_2}{u_1}\int\frac{\Pi \, d t^3}{u_1 \, d\,\frac{u_2}{u_1}}.$$

24. The parallax of the Moon

$$= P\left\{1 + e\cos(c\theta - \alpha) + m^2\cos(\overline{2 - 2m}\,\theta + 2\beta)\right.$$

$$\left. + \frac{15}{8}\,m e \cos(\overline{2 - 2m - c}\,\theta + 2\beta + \alpha)\right\},$$

where $P =$ mean parallax, $m = \frac{\text{Sun's mean motion}}{\text{Moon's mean motion}}$, $\theta =$ longitude of Moon, $\alpha =$ longitude of perigee, $-\beta =$ Sun's mean longitude when $\theta = 0$. Explain fully the effects of the several terms in the above expression on the Moon's orbit.

MORNING PROBLEMS.—Mr. Jeffreys & Mr. Bowstead.

1. Transform the equation $x^3 - p x^2 + q x - r = 0$, whose roots are a, b, c, into one whose roots are

$$\frac{c}{a+b-c}, \frac{b}{a+c-b}, \frac{a}{b+c-a}.$$

2. If I have 9 half-guineas and 6 half-crowns in my purse, how may I pay a debt of 4£. 11*s*. 6*d*.?

3. If from two fixed points in the circumference of a circle, straight lines be drawn intercepting a given arc and meeting without the circle, the locus of their intersections is a circle.

4. Given the equations of two straight lines, find the equation to a third which shall pass through their point of intersection and make equal angles with them; and shew from the result that there are two straight lines, at right angles to each other, which satisfy the question.

5. If three parallel forces acting at the angular points A, B, C of a plane triangle are respectively proportional to the opposite sides a, b, c; prove that the distance of the centre of parallel forces from A

$$= \frac{2bc}{a+b+c} \cos \frac{A}{2}.$$

6. In the ellipse if the distances CP, CQ be drawn at right angles to each other, prove that

$$\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{AC^2} + \frac{1}{BC^2}.$$

7. If R, r be the radii of the circumscribed and inscribed circles of a regular polygon of m sides, and R', r' the corresponding radii for a regular polygon of $2m$ sides and of the same perimeter as the former, then $Rr' = R'^2$ and $R + r = 2r'$.

8. The chord of curvature at any point (x, y) of a curve, drawn through a point whose co-ordinates are α, β,

$$= \frac{2(1+p^2)\left\{y-\beta-p(x-\alpha)\right\}}{q\sqrt{(x-\alpha)^2+(y-\beta)^2}}, \text{ where } p = \frac{dy}{dx}, q = \frac{d^2y}{dx^2}.$$

9. A body acted on by a centripetal force varying partly as $\frac{1}{D^3}$ and partly as $\frac{1}{D^5}$, is projected with the velocity which would be acquired in falling from infinity, at an angle with the distance, whose tangent $= \sqrt{2}$, the forces being equal at the point of projection. Required the orbit described, and the time of descent to the centre.

10. An imperfectly elastic body slides down a smooth plane of given length, and is reflected from the horizontal plane. Find the inclination of the plane that the range may be a maximum, and find the range.

11. If b_n represent the co-efficient of x^n in the expansion of any function of x by *Maclaurin's* Theorem, and a_n in a similar expansion of the hyperbolic logarithm of that function, prove that

$$a_n = \frac{1}{n\,b_0}\left\{-(n-1)b_1 a_{n-1} - (n-2)b_2 a_{n-2} - \&c. - b_{n-1}a_1 + nb_n\right\},$$

and apply this theorem to determine the relation between the co-efficients in the expansion of hyp. log cos x.

12. The shadow cast by an oblate spheroid resting on its vertex in the Sun, on a horizontal plane, is an ellipse; and the spheroid stands in its focus.

13. Integrate

$$\frac{dx}{(x+2)^2(1+x^2)},\quad \frac{dx}{x^3\sqrt[3]{a^3-x^3}};\ \text{and if}\ \frac{dy}{dx}=1+xy,$$

shew that

$$y=1+\frac{1}{1\,.\,3}+\frac{1}{1\,.\,3\,.\,5}+\&\text{c. between } x=0, \text{ and } x=1:$$

also, integrate $\dfrac{z+y}{y}\,dx-\dfrac{x+y}{z}\,dz-\dfrac{y^2-xz}{y^2}\,dy=0$, by applying the criterion of integrability.

14. What is the least depth of fluid, in which a given cone can rest permanently with its axis vertical, the vertex of the cone resting on the base of the vessel, and the specific gravities of the cone and fluid being given.

15. If at a place between the tropics, c be the zenith distance of a known star, when the ecliptic comes upon the zenith of the place of observation, and θ the longitude of the Earth, when the corresponding aberration in zenith distance vanishes, prove that θ is determined by the equation

$$\cot(\theta-L)=\frac{\sin^2\lambda\cos c}{\sqrt{\sin(c+\lambda)\sin(c-\lambda)}},$$

where L and λ represent the longitude and latitude of the star.

16. Find the ratio of the height to the diameter of the base of a cylinder, that the moment of inertia may be the same about any axis whatever passing through its centre of gravity.

17. Find the length of the tide-day, the Sun and Moon being in the equator, and shew how the densities of the Sun and Moon may be compared, by observing the lengths of the greatest and least tide-days.

18. State the general nature of developable surfaces. Investigate the partial differential equation of the second order, which belongs to them, and integrate for the partial differentials of the first order. Shew also how such surfaces may be obtained from given curves of double curvature.

MONDAY AFTERNOON.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

First and Second Classes.

1. Trace the curve of which the equation is $y^2(x-a) = x^3 - b^3$, when $a > b$ and when $a < b$. Find its asymptotes and singular points.

2. Integrate the differentials $e^x \sin^m x\,dx$, and the differential equation

$$x^2\,dy = (x^2 - ay^2)\,dx.$$

3. Draw a tangent plane to any curve surface, and determine the angle it makes with the plane of xy.

4. Find Σx^n in a series proceeding according to the descending powers of x.

5. In a common pump find the height through which the water ascends at any stroke.

6. A spherical surface being constituted of particles the forces of which vary as $\frac{1}{D^2}$; shew that the attraction of the whole surface on a particle without it, varies inversely as the square of the distance of the particle from the centre. (Newton, Book I. Prop. 71.)

7. If a body oscillate in a medium in which the resistance varies as the square of the velocity; the differences between the times of oscillation in the medium and in vacuo are proportional to the arcs nearly. (Newton, Book II. Prop. 27.)

8. The elevation of the summit of the spheroid produced by the attraction of the Sun and Moon on a fluid sphere is double of the depression of the equator below the sphere.

9. Investigate the motion of the pole of the Earth produced by the Moon in one sidereal revolution.

10. If V be any function of x, y, p, q, &c. prove that when $\int V d x$ is a maximum or minimum,

$$N - \frac{d P}{d x} + \frac{d^2 Q}{d x^2} - \text{\&c.} = 0.$$

MONDAY AFTERNOON.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

Third and Fourth Classes.

1. Having given the equation to a plane, find the equations to a straight line perpendicular to it, and passing through a given point.

2. If the observed angular distance of two points be a, their observed elevations above the horizon being H and h, and θ the angular distance reduced to the horizon, shew that

$$\sin^2 \frac{\theta}{2} = \frac{\sin \frac{1}{2} (a + H - h) \sin \frac{1}{2} (a + h - H)}{\cos H \cos h}.$$

3. Integrate $\dfrac{(x + 2)\, d x}{(x^2 + 1)(x - 3)^2}$, and the following differential equation $d y + y\, d x = a x^3 d x$.

4. Explain the method of summing any recurring series, and sum the series 2, 5, 13, 35, &c. to n terms.

5. A body being acted upon by any number of forces in the same plane; find the equations of equilibrium.

6. When a body is acted upon by forces X and Y in the directions of the co-ordinates x and y,

$$\text{prove } \frac{d^2 x}{d t^2} = X \text{ and } \frac{d^2 y}{d t^2} = Y.$$

7. Find the specific gravity of a body lighter than the fluid in which it is weighed.

8. In *Newton* Prop. 66. Book I. explain the effect of the disturbing force of S in producing a motion of the nodes of P's orbit, and a variation of the inclination.

9. Find when the altitude of a known star increases fastest.

10. If a small pencil of parallel homogeneal rays be refracted into a sphere; find at what angle the rays must be incident that they may emerge parallel after any given number of reflections within the sphere.

TUESDAY MORNING.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

First Class.

1. Explain *Newton's* rule for discovering impossible roots in any equation.

2. Assuming the ordinary expansion of $\delta \int V dx$, determine the requisite addition to be made to it when V involves the limiting values of x, y, p, &c.; and apply the method to find the position of the curve of quickest descent from one curve to another, when the motion commences from the first curve.

3. Shew that the solution of the equation of differences

$$u_{x+n} + A_x u_{x+n-1} + \&c. + P_x u_x = Q_x,$$

may be made to depend upon the solution of the equation

$$u_{x+n} + A_x u_{x+n-1} + \&c. + P_x u_x = 0.$$

4. Investigate the general form of the equation to cylindrical surfaces, and apply it when the directrix is a curve of which the equations are

$$x^2 + y^2 = r^2 \text{ and } z = cx.$$

5. Apply *Lagrange's* theorem to determine the least root of the equation

$$x^3 - 5x + 7 = 0.$$

6. A body moveable about a fixed axis is acted upon by a single force in a plane perpendicular to the axis: find the pressure on the axis arising from that force, and thence determine fully the co-ordinates of the centre of percussion.

7. Find the horary motion of the Nodes in an elliptic orbit. (Newton, Book III. Prop. 31.)

8. Determine the attraction of an oblate spheroid on a particle situated in its equator.

9. Explain *fully* the method of determining the Sun's parallax by the transit of Venus over the Sun's disk.

TUESDAY MORNING.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

Second and Third Classes.

1. If there be a chances for an event happening and b for its failing in one trial, find the probability of its happening t times *at least* in n trials.

2. In the equation $x^2 - px + q = 0$, find the sum of the n^{th} powers of the roots in terms of the co-efficients.

3. In the general equation of the second degree

$$ay^2 + bxy + cx^2 + ex + fy + g = 0,$$

shew in what cases the curve will be an ellipse, hyperbola and parabola; and find the co-ordinates of the centre in the former case.

4. Investigate the conditions requisite in order that a function of two variables x, y may be a maximum or minimum. Apply them to find when

$$u = x^4 + y^4 - 4axy^2$$

is a maximum or minimum.

5. Shew that in a small spherical triangle, if $\frac{1}{3}$ of the *Spherical Excess* be subtracted from each of the angles, the resulting angles will be those of a plane triangle having the same sides as those of the spherical triangle.

6. When a body moves upon a surface of revolution, find the re-action of the surface.

7. If a body be acted upon by any forces, the motion of the centre of gravity will be the same as if all the forces were applied at that point: and the motion of rotation will be the same as if the centre of gravity were fixed and the same forces applied.

8. Explain accurately what is meant by the *Equation of Time*; and shew that it vanishes four times in a year.

9. Find the horary increment of the area described by the Moon, and compare its values at quadrature and syzygy. (Newton, Book III. Prop. 26.)

10. Define the *Metacentre* of any floating body, and investigate a formula for its position.

TUESDAY MORNING.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

Fourth Class.

1. If a, b, c, &c. be the roots of an equation, find the value of

$$a^2b + a^2c + b^2a + \&c.$$

2. Investigate the polar equation to the hyperbola, the focus being the pole (given that $SP - HP = 2AC$), and draw the asymptote by means of this equation.

3. Trace the curve whose equation is $y = \frac{a^2 x}{a^2 + x^2}$, and find the number and nature of its singular points.

4. Integrate $\frac{dx}{\sqrt[3]{1 + x^3}}$, $\frac{x\,dx}{\sqrt{a + bx + cx^2}}$, $\frac{dx}{\cos^3 x}$.

5. Define the radius of gyration of any body or system of bodies moveable about a fixed axis, and investigate an expression for determining its magnitude: apply also this expression to a sphere revolving about a diameter.

6. Determine by the principles of *Newton's* seventh Section the spaces due to the velocity externally and internally in an ellipse, the force being in the centre.

7. Find the effect of the disturbing force of the Sun on the Apsides of the lunar orbit during one revolution of the Moon. (Newton, Book I. Prop. 66. Cor. 7.)

8. Explain the construction of the Fire engine, and shew the use of the *Air-vessel.*

9. Find the caustic when the reflecting curve is a logarithmic spiral and the luminous point in the pole.

10. Construct a vertical south dial, and determine how much of it must be graduated.

TUESDAY AFTERNOON.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

First Class.

1. Prove that

$$\Delta^n u_x = \frac{d^n u_x}{dx^n} + \frac{\Delta^n 0^{n+1}}{1.2\ldots(n+1)} \frac{d^{n+1} u_x}{dx^{n+1}} + \frac{\Delta^n 0^{n+2}}{1.2\ldots(n+2)} \frac{d^{n+2} u_x}{dx^{n+2}} + \&c.$$

2. In any surface of the second order which has a centre, the sum of the squares of any system of conjugate diameters is equal to the sum of the squares of the principal diameters.

3. Shew under what condition the equation

$$P\,dx + Q\,dy + R\,dz = 0$$

is integrable. Find also the factor which will render it integrable when homogeneous.

4. Explain the method of integrating the partial differential equation $Pp + Qq = R$, when $P\,dy - Q\,dx = 0$ and $P\,dz - R\,dx = 0$ are integrable separately. Apply the method to the equation $py + qx = z$.

5. Find the angles which the axis of instantaneous rotation makes with the co-ordinates x, y, z.

6. If an incompressible fluid contained in any vessel flow through an orifice k with a velocity u, z' being the vertical co-ordinate of the upper surface, and y' its area, z, y the same quantities for any other horizontal section, p the pressure on a unit of this section,

$$p = \Pi + g(z - z') - k\frac{du}{dt}\int\frac{dz}{y} - \frac{k^2u^2}{2}\left(\frac{1}{y^2} - \frac{1}{y'^2}\right),$$

where Π is the atmospheric pressure on the surface. Also, from this expression find the velocity of the issuing fluid when the orifice is small.

7. If a point move through one or more spaces bounded by parallel planes, and be acted upon by a force which is perpendicular to the planes, and which is the same at the same distance from them, the angle of incidence is to the angle of emergence in a given ratio. (Newton, Book I. Prop. 94.)

8. In a homogeneous spheroid attracting a point on the surface, the effect of the force parallel to the equator is as the distance from the axis.

9. Having given three geocentric places of a comet, find the corresponding heliocentric and geocentric distances.

TUESDAY AFTERNOON.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

Second and Third Classes.

1. Find the sum of the m^{th} powers of the roots of an equation in terms of the co-efficients and the sums of the inferior powers.

2. If a curve have as many asymptotes as it has dimensions, and a right line be drawn which cuts them all, the parts of the line measured from the asymptotes to the curve will together be equal to the parts measured in the same direction from the curve to the asymptotes.

3. Shew the method of extracting the cube root of the binomial surd $a + \sqrt{b}$, and apply it to the solution of the equation $x^3 - 3x - 18 = 0$ by *Cardan's* rule.

4. Expand $\dfrac{1}{1 + e \cos x}$ into a series of the form $A + B \cos x + C \cos 2x +$ &c. and explain the law of the co-efficients.

5. Eliminate by differentiation the constants from the equation $y^2 = ax + bx^2$, and shew how many *Derivatives* of the m^{th} order there are to an equation containing n arbitrary constants.

6. Find the horary variation of the inclination of the lunar orbit to the plane of the ecliptic, and thence by *Newton's* construction determine its mean monthly value.

7. Explain the nature and use of the Ballistic Pendulum, and perform the requisite calculations in the experiment.

8. Find the Moon's parallax by observations made out of the plane of the meridian, and shew how the effect of refraction may be avoided.

9. If a body move in a surface of revolution acted upon by a centre of force situated in the axis, the areas described, projected on a plane perpendicular to the axis, are proportional to the times. (Newton, Book I. Prop. 55.)

10. Find the attraction of a homogeneous spheroid of small eccentricity on a particle situated in its pole.

TUESDAY AFTERNOON.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

Fourth Class.

1. If a circle be described on AM the axis major of an ellipse, and if an ordinate to the axis meet the ellipse in P and the circle in Q, S being a point in the major axis, the areas ASP, ASQ are in a constant ratio.

2. Solve by *Cardan's* rule $x^3 + 3x^2 + 9x - 13 = 0$.

3. If $5x + 21y = 2000$, find x and y, and the number of positive integer solutions which the equation admits of.

4. Integrate

$$dx\,(a^2 + x^2)^{\frac{5}{2}} \text{ and } d\theta \sin m\theta \cos n\theta \cos p\theta.$$

5. Shew that when a body moves in an inverted cycloid, the force by which it is urged along the curve varies as the arc to the lowest point; and hence shew that the oscillations are isochronous.

6. Construct the Air Pump, and shew that the quantities of air expelled by successive strokes are in geometrical progression.

7. A person can see distinctly at the distance of four inches; find the focal length and nature of a lens which will enable him to see distinctly at the distance of sixteen inches.

8. If the velocities of two bodies at any equal distances from the centre of force be the same, and if one body move

in a straight line to or from the centre and the other in a curve; their velocities will be the same at all other equal distances. (Newton, Book I. Prop. 40.)

9. If S and P revolve round their common centre of gravity by their mutual attraction, shew that each will move in the same manner as if a body were placed in the centre of gravity exerting a force varying according to the same law. What is the magnitude of this body for each of the two S and P, the force varying inversely as the square of the distance?

10. Find the length of the longest day in latitude 52° 13′, the obliquity of the ecliptic being 23° 28′.

Having given $10.1105786 = \log \tan 52° 13'$,

$$9.6376106 = \log \tan 23° 28',$$

$$9.7481230 = \log \cos 55° 57',$$

$$9.7483099 = \log \cos 55° 56'.$$

1830.

MODERATORS,

Mr. Hanson, Caius. Mr. King, Queen's.

EXAMINERS,

Mr. Miller, St. John's. Mr. Bowstead, Corpus.

FRIDAY MORNING.

QUESTIONS IN PURE MATHEMATICS.

First, Second, Third and Fourth Classes.

1. In any right-angled triangle, the square which is described upon the side subtending the right angle, is equal to the sum of the squares described upon the sides containing the right angle.

2. The sides about the equal angles of equiangular triangles are proportional, and those which are opposite to the equal angles are homologous sides.

3. Divide 1532 feet $9\frac{9}{12}$ inches by 81 feet 9 inches.

4. Find the amount of an annuity A in n years, and the present worth of 140£. per annum for ever at 5 per cent.

5. In any number, if a point be placed over every third digit, beginning with the one on the right hand, shew that the

number of digits in the cube root is equal to the number of such points; and extract the cube root of 318.61199 to two places of decimals.

6. Investigate a rule for finding the greatest common measure of any two algebraical quantities, and shew that a factor of any divisor, which is not contained in the corresponding dividend, may be removed without affecting the result.

7. Solve the following equations:

$$(1) \quad \sqrt{x+16} = 2 + \sqrt{x},$$

$$(2) \quad \left.\begin{matrix} x^3 + y^3 = 1001 \\ x \; + y = \;\; 11 \end{matrix}\right\},$$

$$(3) \quad \sqrt[m]{(1+x)^2} - \sqrt[m]{(1-x)^2} = \sqrt[m]{1-x^2}.$$

8. The co-efficients of the terms of an expanded binomial are whole numbers, when the index is a whole number; and the co-efficients of the terms equidistant from the extremes are equal.

9. If two magnitudes, when substituted for the unknown quantity in an equation, give results affected with different signs, an odd number of roots lies between them; but if they give results with the same sign, either no root or an even number of roots lies between them.

10. An equation of m dimensions has n equal roots, shew how to find them: and solve the equation

$$x^4 + 13x^3 + 33x^2 + 31x + 10 = 0,$$

which has three equal roots.

11. Prove that in any system of logarithms

$$\log MN = \log M + \log N, \; \log \frac{M}{N} = \log M - \log N,$$

$$\text{and } \log M^n = n \log M.$$

Explain the advantages of *Briggs's* system, and having a table constructed for one system, give the method of constructing a table for a different system.

12. Find the sines of the sum and difference of two arcs in terms of the sines and cosines of the arcs themselves.

13. Given the sides of a plane triangle, find the cosine of an angle; investigate formulæ adapted to logarithmic computation for the solution of the triangle, and explain which of the methods is best in particular cases.

14. Given $\tan A$, $\tan B \ldots\ldots$ find $\tan (A + B + \ldots\ldots)$ and thence deduce $\tan 7A$.

15. Prove *Napier's* rules for the solution of right-angled triangles when one of the sides is the middle part; and having given one side and an angle opposite to it, solve the triangle and explain whether there is any ambiguity.

16. If the distance of any point P from a fixed point S be in a given ratio to its distance PM from a fixed line, find the rectangular equation to the locus of P, according as SP is equal to, less than or greater than PM.

17. In the ellipse, $\dfrac{PV \cdot VG}{QV^2} = \dfrac{CP^2}{CD^2}$.

18. Investigate a rule for transforming numbers from one scale of notation to another, and transform 42.36 from the denary scale to the scale of 5.

19. In the expansion of $(a + b + c + \ldots\ldots)^m$, find the co-efficients of $a^p\, b^q\, c^r$ &c.

20. If m be a prime number, and a a number not divisible by m, then $a^{m-1} - 1$ is divisible by m.

21. Find the equation to a straight line passing through a given point, and cutting a given straight line at a given angle. Required also the co-ordinates of the point of intersection.

FRIDAY AFTERNOON.

QUESTIONS IN NATURAL PHILOSOPHY.

First, Second, Third and Fourth Classes.

1. A cord passing round a fixed point is drawn in different directions by two equal forces acting at a given angle; find the pressure on the point.

2. Explain the method of graduating the common steelyard.

3. Find the relation between the power and the weight when there is an equilibrium on the inclined plane.

4. Two bodies whose common elasticity is e, moving with given velocities, impinge directly upon each other: it is required to determine their velocities after impact.

5. Define *Specific Gravity:* what is the relation between the weight, volume and specific gravity of any substance? Explain the meaning of the numbers given in tables of specific gravity.

6. Describe *Nicholson's* Hydrometer, and shew how the specific gravity of a fluid may be found by it.

7. Explain the experiment by which it is ascertained that the density of air is proportional to the force which compresses it.

8. Describe the common pump, and find the least play of the piston which will enable the pump to work, the lower valve being at the surface of the water.

9. Having given the position of an object placed between two plane mirrors inclined to each other at a given angle, find the number and positions of the images.

10. Diverging rays are incident nearly perpendicularly upon a given spherical refracting surface; having given the focus of incidence, find the focus after refraction, and prove that the conjugate foci move in the same direction upon the axis of the surface.

11. Explain the construction of the divided object glass Micrometer, and shew its use.

12. Enunciate and prove *Newton*, Lemma 9.

13. A body is projected round a centre of force varying as the distance, with a given velocity, in a given direction; find the magnitudes and positions of the axes of the orbit described, and also the periodic time. (Newton, Book I. Prop. 10. Cors. 1 and 2.)

14. A body revolves in a parabola, find the law of the force tending to the focus. (Newton, Book I. Prop. 13.)

15. In different conic sections described round the same centre of force, situated in the focus, the latera recta are as the squares of the areas described in a given time. (Newton, Book I. Prop. 14.)

16. Explain what is meant by the *Error of the line of Collimation*, and shew how it may be avoided.

17. Having given the sidereal time of any phænomenon, find the corresponding mean solar time.

18. Find the latitude of the place of observation, from two equal altitudes of the Sun before and after noon, and the time between.

19. Determine the precession in north polar distance and right ascension, and shew when it is additive and when subtractive.

EVENING PROBLEMS.—Mr. Hanson.

1. Find the value of $\frac{1}{(\sin\theta)^2} - \frac{1}{\theta^2}$, when $\theta = 0$.

2. A person spends in the first year m times the interest of his property; in the second $2m$ times that of the remainder; in the third $3m$ times that at the end of the second, and so on; and at the end of $2p$ years he has nothing remaining; shew that in the p^{th} year he spends as much as he has left at the end of that year.

3. Given
$$\left.\begin{aligned}\tan\theta + \tan\phi + \tan\psi &= 1 + \frac{4}{\sqrt{3}},\\ \tan\theta\tan\phi + \tan\theta\tan\psi + \tan\phi\tan\psi &= 1 + \frac{4}{\sqrt{3}},\\ \tan\theta\tan\phi\tan\psi &= 1,\end{aligned}\right\}$$
find θ, ϕ and ψ; and sum the series
$$(\sec\theta)^2 + \left(\frac{1}{2}\sec\frac{\theta}{2}\right)^2 + \left(\frac{1}{2^2}\sec\frac{\theta}{2^2}\right)^2 + \left(\frac{1}{2^3}\sec\frac{\theta}{2^3}\right)^2 + \text{..} \textit{ad inf.}$$

4. In any polygon with n sides $A_1 A_2$, $A_2 A_3$, ... respectively represented by a_1, a_2 .... prove that
$$a_1 \sin A_1 - a_2 \sin(A_1 + A_2) + a_3 \sin(A_1 + A_2 + A_3) - \ldots\ldots$$
$$\pm\, a_{n-1} \sin(A_1 + A_2 + A_3 + \ldots\ldots + A_{n-1}) = 0.$$

5. A ray of light is refracted through a prism, the angle of which is 60° and index of refraction $\sqrt{2}$, so as to undergo the least possible deviation; determine that deviation. Shew also that no ray can be directly transmitted through a prism of the same refracting power when the angle exceeds 90°.

6. If $\left.\begin{aligned} aX + bY + cZ &= 0\\ a_1X + b_1Y + c_1Z &= 0\end{aligned}\right\}$ where $\begin{aligned} X &= ax + a_1x_1 + a_2,\\ Y &= bx + b_1x_1 + b_2,\\ Z &= cx + c_1x_1 + c_2,\end{aligned}$

then $X^2 + Y^2 + Z^2$
$$= \frac{\{a_2(bc_1 - b_1c) + b_2(a_1c - ac_1) + c_2(ab_1 - a_1b)\}^2}{(bc_1 - b_1c)^2 + (a_1c - ac_1)^2 + (ab_1 - a_1b)^2}.$$

7. Trace the curve the equation to which is $y = e^{\sin x}$, and express in a series the area which recurs.

8. A perfectly smooth rod in a vertical plane revolves uniformly round a vertical axis, and a ring placed on it is attracted to a horizontal plane by a force varying as the distance in addition to the uniform force of gravity; required the form of the rod that the ring may remain on whatever point it is placed.

9. An ellipse may be constructed so that if any abscissa be taken to represent the aberration in longitude of a given star, the corresponding ordinate will represent the aberration in latitude, co-ordinates being measured from the centre along the axes; prove this and determine the axes.

10. The equation to the path of a projectile is

$$y = a x + \frac{g}{k^2} \text{hyp. log}\,(1 - b x),$$

gravity ($= g$) acting parallel to the axis of y; shew that the resistance $= k$ velocity.

11. Find the volume of a solid the equation to which is

$$z = e^{-\frac{y}{a}(a^2 + x^2)}$$

$$\text{between } \left\{ \begin{matrix} x = 0 \\ x = \alpha \end{matrix} \right\} \text{ and } \left\{ \begin{matrix} y = 0 \\ y = \infty \end{matrix} \right\},$$

$$\text{and integrate } \frac{d x}{(1 - x^{\frac{2}{3}})^{\frac{3}{2}}}, \; d\,\theta\, \frac{a + b \tan\theta}{A + B \tan\theta},$$

$$\frac{d y}{d x}\,\frac{d^2 y}{d x^2} + (2 x + a) \frac{d^2 y}{d x^2} + 2 \frac{d y}{d x} + 4 x + 2 a = 0.$$

12. A circular sector revolves through any angle round one of its extreme radii; find the centre of gravity of the solid generated, its density varying as the n^{th} power of the distance from the centre of the circle.

13. In the above case, supposing the angle of the sector and the angle through which it has revolved to remain the same, prove that as the radius varies the motion of the centre of gravity will be in a plane passing through the centre of the circle; find the line of motion and the equation to the plane.

14. Water issues from the horizontal surface of a fountain, at an angle a, with a velocity due to h, through a circular annulus of which the radius is r: V is the volume contained by the surface of the fountain, the ascending and the descending

stream; and V' by the surface, the ascending stream and a plane touching it at the highest point: prove that $\frac{V}{V'}$ is constant when $\sin 2\alpha \propto \frac{r}{h}$.

15. A body acted on by gravity oscillates in a curve, and a chain of given length, suspended from the horizontal ordinate where the motion commences, is divided by the ordinate at each point in two parts proportional to the two parts of the tension at that point arising from the centrifugal force and from gravity. What is the curve?

16. A paraboloid revolving round its axis strikes a body P in a direction perpendicular to the radius, and P, being attracted to the intersection of the radius and axis by a force varying as $\frac{1}{D^2}$, after impact describes a parabola of the same dimensions as the generating one. Determine the velocity of rotation and the point of impact.

17. A given opaque sphere and a given luminous paraboloid of revolution have their axes in the same line; the distance between them being known, deduce the equation to the surface of the shadow, and find the form of the shadow thrown on a given plane.

18. In the series of quantities $A_1, A_2, A_3, \ldots$

$$\text{if } A_1 = r \tan\left(\overline{\sin \frac{2\pi}{3} + \alpha}\right), \quad A_2 = r \tan\left(\overline{\sin \frac{4\pi}{3} + \alpha}\right),$$

and the remaining ones be derived according to the following law:

$$A_1 . A_2 . A_3 = r^2 (A_1 + A_2 + A_3),$$
$$A_2 . A_3 . A_4 = r^2 (A_2 + A_3 + A_4), \ldots$$

$$\text{prove that } A_n = r \tan\left(\overline{\sin \frac{2n\pi}{3} + \alpha}\right).$$

19. If $A y^3 + B x y^2 + C x y + D x^2 + E y + F x = 0$ be the equation to a curve; $B y^2 + C y + 2 D x + F = 0$ is the equation to a parabola which bisects all the chords parallel to the axis of x.

$2 B y^2 + C y + 2 D x + F = 0$ and $C y + 2 D x + F = 0$ are equations to a parabola and straight line which are asymptotes to the curve.

The two parabolas and the straight line have a common point of contact in the bisection of that chord which passes through the origin.

20. If a pendulum of length l vibrate in a small circular arc in a medium of which the resistance $= kv^2$ to velocity v, and if s be the arc described from the commencement of a vibration to the point where the velocity is greatest when the friction at the axis of suspension is taken into account, and s' the corresponding arc when the friction is neglected, prove that

$$e^{2ks'} - e^{2ks} = f \cdot \frac{2kl}{g},$$

f being the constant effect of friction, and g gravity.

21. Two Planets P_1, P_2 revolve in circular orbits at the distances r_1, r_2 from the Sun, and when they appear stationary to one another cot P_2's elongation seen from $P_1 = \frac{1}{2} \tan \theta$; shew that $\frac{r_1}{r_2} = \frac{1}{2} \tan \frac{\theta}{2} \tan \theta$.

22. $\left.\begin{array}{l} A x + B y + C z = 0 \\ A' x + B' y + C' z = 0 \end{array}\right\}$ are the equations to the planes in which two planets move. Apply them to find the inclination of the orbits to one another, in terms of their inclinations to the ecliptic and of the longitudes of their ascending nodes, the ecliptic being in the plane of x and y.

23. If the Earth be an oblate spheroid of small ellipticity with semi-axes a and b, the ratio of the mean density to that at the surface is

$$\frac{3}{k^2 a^2}\left(1 - \frac{k(4a - b)}{3 \tan k b}\right) \text{ very nearly,}$$

assuming the density to be uniform throughout each spheroidal stratum at the same distance from the Earth's surface, and to vary as $\frac{\sin k r}{r}$ at different distances, where k is a constant quantity and r the polar semi-axis of the surface of equal density.

24. Explain the theory of the interferences of light, and determine the colour, origin and intensity of a ray resulting from the interference of two similar rays, differing in origin and intensity.

SATURDAY MORNING.

QUESTIONS IN PURE MATHEMATICS.

First, Second, Third and Fourth Classes.

1. Similar triangles are to one another in the duplicate ratio of their homologous sides.

2. If two straight lines meeting one another, be parallel to two straight lines which meet one another, but are not in the same plane with the first two, the plane which passes through these, is parallel to the plane passing through the others.

3. If $y = ax + b$, and $y = a'x + b'$, be the equations to two straight lines in the same plane; prove that the cosine of the angle contained between them is equal to

$$\frac{1 + a a'}{\sqrt{(1 + a^2)(1 + a'^2)}}.$$

4. If $\frac{A_1}{B_1}$, $\frac{A_2}{B_2}$, &c. $\frac{A_n}{B_n}$ be a series of fractions approximating to $\frac{A}{B}$, shew that

$$\frac{A}{B} \backsim \frac{A_n}{B_n} < \frac{1}{B_n^2} \text{ and } > \frac{1}{B_n(B_n + B_{n+1})}.$$

5. Resolve $x^m - 1$ into its simple and quadratic factors.

6. In a spherical triangle,

$$\cot a \sin b = \cos b \cos C + \sin C \cot A.$$

7. If CP and CD be semi-conjugate diameters of an hyperbola, prove that

$$CP^2 - CD^2 = AC^2 - BC^2.$$

8. Investigate *Napier's* analogies: shew for what cases in the solution of spherical triangles they are applicable; shew also how these cases may be solved by the aid of *Napier's* rules alone.

9. Shew how to find the number corresponding to a given logarithm not found exactly in the tables.

10. If a, b and c be three whole numbers taken in succession, prove that Nap. log b

$$= \tfrac{1}{2}\,\text{Nap. log}\, a + \tfrac{1}{2}\,\text{Nap. log}\, c + \left(\frac{1}{2ac+1} + \frac{1}{3}\frac{1}{(2ac+1)^3} + \&c.\right);$$

and shew the peculiar use of this formula in finding the logarithms of prime numbers.

11. From the equation

$$y^3 - ay + x = 0,$$

find y in a series ascending by the powers of x, by the *Reversion of Series*.

12. Having given the equation to a plane, and the co-ordinates of a given point without it, find the equations to a straight line drawn from the point perpendicular to the plane; and determine its length.

13. Define the Differential Co-efficient of any function, and from that definition find the differential co-efficient of uv, u and v being functions of x.

14. Find the differential of the surface of a solid of revolution.

15. Prove *Maclaurin's* theorem, and thence expand $\tan^{-1} x$ to 5 terms.

16. Define the Radius of Curvature, and shew that in curves referred to rectangular co-ordinates it $= \dfrac{\left(1 + \dfrac{dy^2}{dx^2}\right)^{\frac{3}{2}}}{-\dfrac{d^2 y}{dx^2}}$: shew also that in general the circle of curvature at once *touches* and *cuts* the curve.

17. Integrate the following differentials:

$$\frac{dx}{x^3(x^2+4)}, \quad \frac{x^5\, dx}{\sqrt{1-x^2}}, \quad \sqrt{x}\, dx \sqrt{1-x^3}, \quad \frac{dx}{\cos x}, \quad e^x x^m\, dx.$$

18. Trace the curve whose equation is

$$a^2 y = x^3 - \frac{b^4}{x},$$

and determine the number and nature of its singular points.

SATURDAY AFTERNOON.

QUESTIONS IN NATURAL PHILOSOPHY.

First, Second, Third and Fourth Classes.

1. Find the centre of gravity of any system of points whatever.

2. Find the resultant of any number of forces acting in the same plane upon a rigid body, and the equation to the line in which it acts.

3. Investigate the equation to the catenary between the arc and abscissa; and shew that the tension at the vertex is equal to the weight of a portion of the catenary of the same length as the radius of curvature at the vertex.

4. If two bodies A and B of elasticity e impinge directly on each other with velocities a and b respectively, and if u and v be their respective velocities after impact, and p and q the velocities lost and gained respectively,

$$\text{then } A\,a^2 + B\,b^2 = A\,u^2 + B\,v^2 + \frac{1-e}{1+e}\,(A\,p^2 + B\,q^2).$$

5. Define the *Centre of Pressure*, and find it in the case of a semi-parabola immersed in a fluid with its base contiguous to the surface.

6. Determine all the positions of equilibrium of an equilateral triangle floating on a fluid with one angle immersed.

7. The compressing force of the air varying as the density, and the force of gravity varying inversely as the square of the distance from the centre of the Earth, find the relation between the density of the air at any altitude and the density at the Earth's surface.

8. Determine the form of a surface which shall refract a pencil of rays proceeding from a given point accurately to another given point.

9. Explain the formation of the primary rainbow, and shew that the breadth of the bow = radius of the red arc − radius of the violet arc + the apparent diameter of the Sun's disk.

10. Construct the astronomical telescope, and having given the focal lengths of the object and eye glasses, find the position of the eye when the field of view is the greatest.

11. When a body descends from a point A towards a centre of force S, the force varying as the distance, shew that the space described, the velocity and the time of motion, are respectively proportional to the versed sine, sine and arc of the circle of radius SA; and find the time to the centre.

12. Explain *Newton's* method of finding the angle between the apsides in orbits nearly circular, and apply it when the force $= m A + \frac{m_1}{A^2}$.

13. *Newton*, Section XI. Prop. 66.

14. In the case of the Sun, Moon and Earth, find the whole force on the Moon in the direction of the radius vector, the orbits being considered in the same plane.

15. Shew that the centres of oscillation and suspension are reciprocal, and explain the use of this property in finding practically the length of a pendulum.

16. Explain fully the *Equation of Time*, and shew at what seasons that part of it arising from the obliquity of the ecliptic is positive, and at what seasons negative.

17. Shew that the inclination of the ecliptic to the horizon is a minimum when Aries rises, and a maximum when it sets; and explain the phenomenon of the Harvest Moon.

EVENING PROBLEMS.—Mr. King.

1. If $x = m \tan (z - nx)$ where x is small compared with z,

$$\text{prove that } x = \frac{m}{2} \frac{\sin 2z}{mn + \cos^2 z} \text{ very nearly.}$$

2. If $- P_{m-p}\, x^{m-p},\ - P_{m-q}\, x^{m-q},\ - P_{m-r}\, x^{m-r},\ -$ &c. be the negative terms of an equation of m dimensions, then will the greatest root of this equation be less than the sum of the two greatest of the quantities $P_{m-p}^{\frac{1}{p}}$, $P_{m-q}^{\frac{1}{q}}$, &c.

3. a and b are respectively the first term and common difference of an arithmetic series,

S_n the sum of n terms,

S_{n+1} .... $(n+1)$ terms,

&c. &c.

prove that $S_n + S_{n+1} + S_{n+2} +$ &c. to n terms

$$= (3n-1)\,n\frac{a}{2} + (7n-2)(n-1)\,n\frac{b}{6}.$$

4. A vertical prismatic column the horizontal section of which is an equilateral and equiangular pentagon is cut by a given plane; find the sides and angles of the section.

5. Eliminate by differentiation $f\left(\frac{y}{x}\right)$ and $\phi\,(x\,y)$ from the equation

$$z = x f\left(\frac{y}{x}\right) + \phi\,(y\,x).$$

6. Find the locus of the intersections of the tangents of an hyperbola with the perpendiculars upon them from the centre: determine its maximum ordinate, its area, and the angles at which it intersects the axis.

7. Having given the first two terms of the expansion of $(a^2 + b^2 + 2\,a\,b\cos\theta)^{-\frac{1}{2}}$ in a series of the form

$$A_0 + A_1\cos\theta + A_2\cos 2\,\theta + \text{\&c.}$$

shew how from them the first two terms of the expansion of $(a^2 + b^2 + 2\,a\,b\cos\theta)^{-\frac{3}{2}}$ may be determined.

8. If S_1 represent the sum of the ordinates in the quadrant of a circle whose radius is 1,

S_2 represent the sum of their squares,

S_3 .... cubes,

&c. &c.

prove that $S_{n-1}\,S_n = \frac{3}{n+1}\,S_1\,S_2$.

9. A straight line revolving in its own plane about a given point intersects a curve line in two points; find the curve when the rectangle of the lines intercepted between the given point and the points of intersection is constant.

10. Prove that if the tangent plane to any curve surface make with the three co-ordinate planes the least possible volume, the distance of the intersections of the plane and axes from the origin are respectively $3x$, $3y$ and $3z$, x, y and z being the co-ordinates of the point of contact.

11. A plane is so moved as always to cut off from a given paraboloid of revolution equal volumes; determine the equation to the surface to which it is always a tangent.

12. Integrate the following differentials and differential equations:

$$\frac{dx}{x^4+1}, \quad \frac{dx}{x^4\sqrt{1-x^2}}, \quad \frac{dx}{\sqrt{a-x}-\sqrt{x}},$$

$$(x^2+y^2)\,dx + x^2y\,dy = 0, \quad (1+x)\frac{d^2y}{dx^2} + a\frac{dy}{dx} = 0,$$

$$1 + p^2 + q^2 = m^2,$$

and also the following equations of differences:

$$f(x^2) - f(x) = m \quad \text{and} \quad u_x\, u_{\pi+x} = k^2.$$

✓ 13. There are two urns A and B, the former containing three white and the latter three black balls: a ball is taken from each at the same time and put into the other, and this operation is repeated three times; what is the probability that A will contain three black and B three white balls?

14. A uniform rod rests with one of its extremities in a semi-circle whose axis is vertical, find the nature of the line supporting its other extremity so that it may rest in every position.

15. Having given the variation of the obliquity of the ecliptic, find the corresponding variations in right ascension and declination.

16. Determine the latitude of the place of observation from observing the times of the rising of two known stars.

17. The axis of a given cone filled with fluid is inclined at a given angle to the horizon; find how much of the fluid will flow out and determine the pressure exercised by the remainder upon the conical surface.

18. A body attracting with a force varying directly as the distance moves uniformly in a straight line; determine the motion of another body situated in the same plane and subject to its influence.

19. Determine the orbit described and the time of describing any angle when a body is projected round a centre of force varying as $\frac{1}{D^7}$, at an angle whose tangent $= \frac{3^{\frac{1}{2}}}{2^{\frac{5}{6}}}$, and with a velocity which is to the velocity in a circle at the same distance $:: \sqrt{2} : \sqrt{3}$.

20. A body descends down the arc of a vertical catenary having its vertex at the lowest point; find the curve of ascent when the oscillations are isochronous, the two curves being so united at the lowest point as to have a common tangent.

21. A corpuscle is attracted by two straight lines at right angles to each other, the particles of which attract with forces varying as $\frac{1}{D^2}$: having given the position of the corpuscle and the length of one of the lines, find the length of the other when the direction in which the corpuscle begins to move is towards their common intersection.

22. A body descends in a straight line in a medium whereof the density varies as the square root of the distance from a given point, and is urged by a constant force tending to that point: find the velocity and time corresponding to a given space, supposing the resistance to vary as the density and velocity jointly.

23. A body describes a circle of given radius uniformly, acted upon by two forces each varying as the distance and without the plane of the circle; find the velocity of the body and the position of the plane of its orbit.

24. If a body revolve in an ellipse round the focus, prove that a progressive motion of the apse will be the effect of any continual addition of force in the direction of the radius vector during the progress of the body from the higher to the lower apse, and point out the effect on the eccentricity.

25. Two balls connected together by an inflexible and inextensible line are constrained to move the one on a horizontal plane, the other on an inclined plane which is at liberty to move freely on the horizontal plane; find the motions of the balls and of the plane, supposing the motion of the rod to be in a vertical plane.

MORNING PROBLEMS.—Mr. Hanson and Mr. King.

1. If n be a whole number, prove that $\dfrac{n^3 + 5n}{6}$ is also a whole number.

2. The ratio between the area of an equilateral and equiangular decagon described about a circle, and that of another within the same circle is equal to $\dfrac{8}{7 + \dfrac{1}{4 + \dfrac{1}{4 + \dots}}}$.

3. If a and b be the sides of a plane triangle, A and B their opposite angles, then will

$$\text{hyp. log } b - \text{hyp. log } a = \cos 2A - \cos 2B$$
$$+ \tfrac{1}{2}(\cos 4A - \cos 4B) + \tfrac{1}{3}(\cos 6A - \cos 6B) + \dots\dots$$

4. Of all spherical triangles which have the same base and equal perpendiculars from the vertex to the base, shew that the isosceles has the greatest vertical angle; and from the result prove that the same is true in plane triangles.

5. Having given

$$\log 8801 = 3.9445320,$$
$$\log 8802 = 3.9445814,$$
$$\log 8804 = 3.9446800,$$
$$\log 8805 = 3.9447294;$$

find log 8803.

6. Two straight lines, which are always tangents to a given parabola, are so inclined to the axis of x that the sum of the co-tangents of the angles which they make with that axis is constant; prove that the locus of their intersections is a straight line parallel to the axis.

7. Find the surface in which the tangent plane always cuts the axis of z at distances from the origin proportional to $\frac{1}{z^n}$; and when $n = 1$ give to the arbitrary function that particular form which will produce the equation to the ellipsoid.

8.
$$\left.\begin{aligned} x &= a z \\ y &= b z \end{aligned}\right\} \quad \text{and} \quad \left.\begin{aligned} x^2 + y^2 &= 2 c x \\ x^2 + y^2 &= m^2 z^2 \end{aligned}\right\}$$

are the equations to a straight line and curve of double curvature; find the equation to the surface generated by a straight line moving always parallel to the plane of xy, and passing through the straight line and the curve.

9. Integrate

$$\frac{d^2 y}{d x^2} + a^2 y = e^x \cos a x,$$

$$z - p x - q y = m (x + y + z).$$

10. Solve the following equations of differences:

$$\Delta^3 x + \Delta^2 x + \Delta x = x^3,$$

$$u_x u_{x+1} + u_x u_{x+2} + u_{x+1} u_{x+2} = m^2.$$

11. Sum the following series:

$$\frac{1}{1\,.\,3}+\frac{1}{4\,.\,6}+\frac{1}{7\,.\,9}+\frac{1}{10\,.\,12}+\ldots\ldots \textit{ad infinitum}:$$

$$\sec\theta\cos\theta+4(\sec\theta)^2\cos 2\theta+13(\sec\theta)^3\cos 3\theta$$
$$+\,40(\sec\theta)^4\cos 4\theta+\text{\&c. to } n \text{ terms.}$$

12. Find $\sin x$ from the equation

$$\sin x\cos x+a\sin^2 x=b,$$

and shew its use in the solution of the following problem: to determine how much the azimuth of a known star on the horizon is affected by refraction.

13. A right-angled triangle vibrates in its own plane about an axis passing through its vertex, find the length of the isochronous simple pendulum; and if one of the sides be slightly diminished and the other as much increased, determine the variation of the pendulum.

14. If a hemisphere and paraboloid of equal bases and similar materials have their bases cemented together, the whole solid will rest on a horizontal plane on any point of the spherical surface if the altitude of the paraboloid $= a\sqrt{\dfrac{3}{2}}$, a being the radius of the hemisphere.

15. Prove that the eye cannot be achromatic for objects at all distances.

16. A body is acted on by two forces, the one repulsive and varying as the distance from a given point, the other constant and acting in parallel lines. Determine the motion of the body.

17. A body falls towards a centre of force which varies as $\dfrac{1}{D^3}$, in a medium of which the density varies as $\dfrac{1}{D^3}$, and the resistance varies as (velocity)^2. Prove that at any distance r from the centre,

$$(\text{velocity})^2 = \frac{m}{h}\left\{1 - e^{-h\left(\frac{1}{r^2} - \frac{1}{a^2}\right)}\right\},$$

where $m =$ force at distance 1, $h =$ density at distance 1 and $a =$ distance from centre at the beginning of the motion.

18. A uniform rod vibrates in a medium the resistance of which varies as the velocity; find the time of one of its small oscillations.

MONDAY AFTERNOON.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

First and Second Classes.

1. Expand $f(x + h, y + k)$ in a series ascending by powers of h and k.

2. Reduce $\dfrac{u_{x+1}}{u_x\, u_{x+2}\, u_{x+3}}$ to an integrable form when $u_x = a + bx$: and sum the series

$$\frac{2^2}{1.3.4.5} + \frac{3^2}{2.4.5.6} + \frac{4^2}{3.5.6.7} + \ldots\ldots \text{ to } n \text{ terms.}$$

3. Prove *Lagrange's* Theorem.

4. A bag contains m white balls and n black balls; find the probability of taking out a white ball at least p times in r trials, the ball being replaced after each trial.

5. State the general nature of developable surfaces. Investigate the partial differential equation of the second order which belongs to them.

6. Find the longitude of the perihelion and the time of the Earth's passing through it.

7. Prove that the centre of gravity of the Earth and Moon describes about the Sun very nearly an ellipse in one plane, and that the area described by its radius vector is very nearly proportional to the time.

8. Find the horary variation of the inclination of the Moon's orbit. (Newton, Book III. Prop. 34.)

9. Find the attraction of an oblate spheroid on a particle in its equator.

10. Having given the declination of the Moon, compare the magnitudes and durations of the superior and inferior tides in any latitude, the effect of the Sun on the tide being neglected.

MONDAY AFTERNOON.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

Third and Fourth Classes.

1. Find the equation to a plane, and determine the constants when the distances of the intersections of the plane with the co-ordinate axes from the origin are given.

2. In a spherical triangle, the sides of which are small compared with the radius of the sphere, having given two sides and the included angle, find the angle between the chords of those two sides.

3. Explain *Newton's* method of approximating to the roots of an equation, and shew that its accuracy does not depend upon the ratio of the quantity assumed to the root, but upon its being nearer to one root than to any other.

4. Integrate $\frac{x^4\,dx}{\sqrt{(2ax - x^2)}}$ between the limits $x = 0$ and $x = a$;

$$\frac{dx}{x^3+1}, \quad (7x + 5y + 3)\frac{dy}{dx} + 28x + 20y - 7 = 0.$$

5. *A* body moving on a curve is acted on by forces X and Y parallel to the axes of the curve, find the reaction; and apply it to find the tension of a string, at the lowest point, when a body oscillates in a circle through an arc of 120°.

6. As the line of nodes of the Moon's orbit moves from syzygy to quadrature, the inclination of the orbit to the ecliptic is diminished: and as the line of nodes moves from quadrature to syzygy, the inclination is increased. (Newton, Book I. Prop. 66. Cor. 10.)

7. A ladder rests with one end on a smooth horizontal plane and the other against a smooth vertical wall; find the horizontal force at its foot which will keep it at rest; and when the force is removed determine its motion.

8. Find experimentally the refracting power of any transparent substance.

9. Find the Sun's right ascension by *Flamstead's* method. Why must the observations be made near an equinox?

10. Explain *Mercator's* projection of the sphere, and find the length of the projection of an arc of the meridian included between the latitudes of 30° and 60°.

TUESDAY MORNING.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

First Class.

1. If from a point two straight lines be drawn and their extremities be joined by a curve; find its nature when the length is a maximum, the area contained by the two lines and the curve being given.

2. Explain the method of integrating the partial differential equation

$$\frac{d^2 z}{d x^2} + P \frac{d^2 z}{d x d y} + Q \frac{d^2 z}{d y^2} = R,$$

where P, Q, R are functions of x, y, z, $\frac{dz}{dx}$, $\frac{dz}{dy}$.

3. Prove that

$$\Sigma u_x = \int u_x dx - \frac{u_x}{2} + \frac{1}{2.1.2.3}\frac{du_x}{dx} - \frac{1}{6.1.2.3.4.5}\frac{d^3u_x}{dx^3} + \&c.,$$

the co-efficients being the same as those of t in the expansion of $\frac{1}{\epsilon^t - 1}$.

4. Find the general equation to conical surfaces; and if a conical surface be described about a surface of the second order, shew that the curve of contact will be in one plane.

5. Find the horary increment of the area which the Moon describes about the earth in a circular orbit. (Newton, Book III. Prop. 26.)

6. A pile of weight w, is driven by a hammer H impinging with a velocity v, the friction being represented by F, find the motion; and when the velocity given to the pile is small, approximate to the whole space through which it is driven.

7. Find the two parts of solar nutation, and prove that they are connected by the equation to an ellipse, the axes of which are in the ratio of cos I : 1, where I is the obliquity of the ecliptic.

8. A body is acted on by gravity; find the tautochronous curve in a medium in which the resistance varies partly as the velocity and partly as the square of the velocity; and from the result prove that it is a cycloid when the resistance vanishes, or varies as the velocity.

9. $s = k\left\{\sin(g\,\theta - \gamma) + \frac{3m}{8}\sin\overline{(2 - 2m - g)\,\theta + 2\beta + \gamma}\right\}$,

where s = tangent of Moon's latitude, γ = longitude of node, $-\beta$ = the Sun's mean longitude when $\theta = 0$. Explain the effect of these terms, and thence shew that the inclination of the orbit is greatest when the line of nodes is in syzygies, and least when it is in quadratures.

TUESDAY MORNING.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

Second and Third Classes.

1. Find the present value of an annuity of 1£. to be continued during the life of an individual of a given age, allowing compound interest for the money.

2. In the surface whose equation is

$$A z^2 + B y^2 + C x^2 + K x = 0,$$

shew in what cases the surface will be respectively an ellipsoid, hyperboloid, elliptic paraboloid, hyperbolic paraboloid and a paraboloid of revolution.

3. Prove that

$$\Delta^n u_x = u_{x+n} - n\, u_{x+n-1} + \frac{n(n-1)}{1 \,.\, 2} u_{x+n-2} - \&c,$$

and thence shew that

$$1 \,.\, 2 \,.\, 3 \,.\, \&c.\, n = n^n - \frac{n}{1}(n-1)^n + \frac{n(n-1)}{1 \,.\, 2}(n-2)^n - \&c.$$

4. Integrate the following differential equations;

$$\frac{dy}{dx} + y = x y^3, \quad \frac{d^2 y}{dx^2} - n^2 y = \cos m x, \quad m^2 \left(\frac{dy}{dx}\right)^3 = \left(y - x\frac{dy}{dx}\right)^2:$$

explain also the relation which subsists between the particular solution and complete integral of a differential equation.

5. Find the moment of inertia of an ellipse revolving in its own plane about any axis.

6. Determine the centre and diameter of the least circle of chromatic aberration, in a given lens.

7. Explain the method of determining altitudes above the Earth's surface by the barometer, account being taken of the variation of temperature.

8. Find the ratio of the diameters of the lunar orbit, supposing it to have been originally without eccentricity. (Newton, Book III. Prop. 28.)

9. Having given the length of a degree of latitude, and also the length of a degree in a direction perpendicular to the meridian, in a given latitude; find the ellipticity of the Earth.

TUESDAY MORNING.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

Fourth Class.

1. The roots of the equation $x^3 - px^2 + qx - r = 0$ are a, b and c, transform it into one the roots of which are

$$\frac{a}{b} + \frac{b}{a}, \quad \frac{a}{c} + \frac{c}{a}, \quad \frac{b}{c} + \frac{c}{b}.$$

2. In each of the conic sections, the radius of curvature

$$= \frac{(\text{normal})^3}{(\frac{1}{2}\text{ lat. rect.})^2}.$$

3. Find the differential of a solid of revolution, and apply it to find the content of a segment of a paraboloid, the radii of the greater and smaller ends of which are a and b respectively, and the distance between them c.

4. Find $\displaystyle\int \frac{dx}{\sqrt{x(1-x)}}$, $\displaystyle\int \frac{dx}{(1+x^2)^{\frac{3}{2}}}$,

$$\int \frac{A(\sin\theta)^n + B(\cos\theta)^n}{(\cos\theta)^{n+2}}\, d\theta, \quad \int \cos(a + b\theta)\cos(A + B\theta)\, d\theta.$$

5. State the Principle of Virtual Velocities, and prove it when two bodies are in equilibrium on a bent lever.

6. Prove that the image of a straight line placed before a spherical reflector is a conic section, and determine in what cases it is each particular conic section.

7. Find the specific gravity of a body lighter than the fluid in which it is weighed.

8. Shew how to determine whether a planet is a superior or an inferior one; and having given the synodic period of a planet and the length of a year, find the planet's period.

9. Construct a horizontal dial, and find the limits beyond which it is unnecessary to graduate it.

10. If two bodies S and P attract each other mutually, the orbit which P appears to describe about S in motion may be described about S fixed, by the action of the same force. (Newton, Book I. Prop. 58.)

TUESDAY AFTERNOON.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

First Class.

1. At any point in any curve surface, the sections of greatest and least curvature are at right angles to each other.

2. Approximate to the following integrals:

$$d\,x \log(1 + e\cos x), \text{ and } \frac{d^2 y}{d x^2} - a x^m y = 0\,;$$

and solve the following partial differential equation,

$$\frac{d^2 y}{d t^2} = a^2 \frac{d^2 y}{d x^2}.$$

3. Explain fully the mode of applying the Calculus of Variations to cases wherein it is required to determine one function u a maximum or minimum, the value of another function v being given: and exemplify it by finding the curve of quickest descent from one given point to another, the length of the curve being given: shew also how the constants introduced by integration may be determined in this case.

4. When a small pencil of diverging rays is incident obliquely on a concave refracting surface, the ultimate intersection of two refracted rays in a normal plane is determined from the equation

$$\frac{(\cos\phi_1)^2}{v}\,\frac{1}{v}-\frac{(\cos\phi)^2}{\mu}\,\frac{1}{u}=\left(\cos\phi_1-\frac{\cos\phi}{\mu}\right)\frac{1}{r}.$$

5. If a system move in any manner whatever, prove that $\int \Sigma\, m\, v\, d\, s$ is a minimum.

6. Determine the circular orbit of a planet from two observations.

7. Find the mean horary motion of the Nodes of the lunar orbit supposed to be elliptical. (Newton, Book III. Prop. 31.)

8. When the force at the pole of a revolving fluid spheroid is to the force at the equator as the equatoreal radius is to the polar radius, any two canals drawn from any points in the surface and meeting within it, will balance each other.

9. Eliminate t from the differential equations:

$$\frac{d\left(\rho^2\,\frac{d\theta}{dt}\right)}{dt}=T\rho,$$

$$\text{and }\frac{d^2\,(\rho s)}{dt^2}=-S.$$

TUESDAY AFTERNOON.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

Second and Third Classes.

1. Given the two sides and the included angle of a spherical triangle, required its area; and from the expression obtained, find the area of a plane triangle in corresponding terms.

2. Find the magnitudes and positions of the principal axes of the curve of the second order, the equation of which is

$$Ay^2+Bxy+Cx^2+Dy+Ex+F=0.$$

3. Explain the method of finding whether a curve has multiple points, and find the number and nature of the multiple points of the curve the equation of which is

$$y^4 - 2a^2y^2 - 2ax^3 - 3a^2x^2 + a^4 = 0.$$

4. Explain the transformation of the independent variable, and transform the equation

$$\frac{d^2y}{dx^2} - \frac{x}{1-x^2}\,\frac{dy}{dx} + \frac{y}{1-x^2} = 0,$$

where x is the independent variable, into one where θ is the independent variable, θ being equal to $\cos^{-1} x$.

5. Find the conditions of the stable and unstable equilibrium of a floating body; and if it revolve about a horizontal axis, shew that it passes alternately through positions of stable and unstable equilibrium.

6. Find the time of an oscillation in a hypocycloid, the body being acted upon by a force varying as the distance from the centre of the globe.

7. When a chain fixed at two points is acted upon by a central attractive or repulsive force, the tension at any point is inversely as the perpendicular from the centre of force upon the tangent at that point.

8. Find the moment of inertia of a system about any axis passing through the origin of the co-ordinates, and the moment of inertia about any axis in terms of the moments about the *principal axes*.

9. If a body acted upon by gravity be projected in a medium the resistance of which varies as the square of the velocity, find the equation to the curve described; and when the resistance vanishes, shew that it is the equation to a parabola.

TUESDAY AFTERNOON.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

Fourth Class.

1. Investigate *Waring's* Rule for the solution of a biquadratic equation.

2. x, y and X, Y are the co-ordinates of a point referred to two systems of rectangular co-ordinates having a common origin and inclined to each other at a given angle: find the relation subsisting between x, y, X and Y.

3. The radius of curvature of a spiral $= \dfrac{r\,dr}{dp}$.

4. Trace the curve whose equation is $a\,y^2 = \dfrac{x^4}{x-a}$, and find its area.

5. Having given the quantity of air in the tube of a barometer, determine the depression below the standard altitude.

6. Having given the radius of the arc of any colour in the primary rainbow, find the index of refraction, for that colour, out of air into water.

7. The difference of the forces at corresponding points in the fixed and revolving orbits varies inversely as the cube of the distance. (Newton, Book I. Prop. 44.)

8. Compare the axis major of the ellipse apparently described by P round T, with that of the ellipse described by P round T fixed in the same periodic time. (Newton, Book I. Prop. 60.)

9. In a given latitude find the Sun's azimuth, his declination and the time of the day being given; and adapt the trigonometrical formula to logarithmic computation.

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p. 133. no 17. p. 165. no 9

42. no 19. If x be very large then $1.2.3\ldots x = \left(\frac{x}{e}\right)^x \sqrt{2\pi x}$ nearly

193 no. 1. | Chances p. 191. no 13. | P. 131. no 1. | P. 195. no. 12.
131 . . . 1 |
168 . . 20 |

es p. 103, 2. N.B. also (by Whewell & King) p. 134. no 7 ?
p. 90; 5

on p. 81 5. See also